02

The degree of electrification of a body is represented by the electrostatic potential of the charged body. The direction of flow of charge between two charged bodies placed in contact with each other is determined by electrostatic potential.

ELECTROSTATIC POTENTIAL AND CAPACITANCE

|TOPIC 1|

Electrostatic Potential, Electrostatic Potential Difference and Electrostatic Potential Energy

The electrostatic potential at any point in the region of electric field is equal to the amount of work done in bringing a unit positive test charge (without acceleration) from infinity to that point.

$$\therefore \quad \text{Electrostatic potential } (V) = \frac{\text{Work done } (W)}{\text{Charge } (q_0)}$$

It is a scalar quantity. Its SI unit is volt (V) and 1V = 1 J/C and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Note

Electrostatic potential (V) at a point is said to be **one volt**, when one joule of work is done in moving one coulomb of positive charge (without acceleration) from infinity to that point.

Work done ($[W_\infty]_{ext}$) by an external force in bringing (without acceleration) a unit positive charge from infinity to a point is equal to the potential (V) at that point,

i.e. $$V = \frac{[W_\infty]_{ext}}{q_0} = \frac{-[W_\infty]_{elec}}{q_0} \qquad [\because [W_\infty]_{ext} = -[W_\infty]_{elec}]$$

CHAPTER CHECKLIST

- Electrostatic Potential, Electrostatic Potential Difference and Electrostatic Potential Energy
- Dielectric and Capacitance

where, $[W_\infty]_{elec}$ is the work done by the electric field on a charged particle as that particle moves from infinity to a point. A potential (V) can be positive, negative or zero depending on the signs and magnitudes of q and W_∞.

Note Electric potential is state dependent function as electrostatic forces are conservative forces. No work is done in moving a unit positive test charge over a closed path in an electric field.

EXAMPLE |1| Potential at a point P in space is given as 3×10^5 V. Find the work done in bringing a charge of 2×10^{-6} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Sol. Given,

Potential at the point P,

$V = 3\times10^5$ V, charge, $q_0 = 2\times10^{-6}$ C

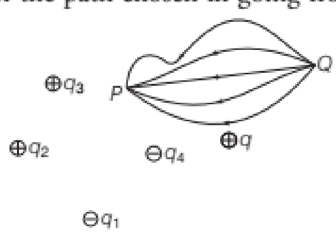
Work done in bringing the charge from infinity to the point P is

$$W_\infty = q_0 V = 2\times10^{-6}\times3\times10^5$$
$$= 6\times10^{-1} = 0.6\text{ J}$$

No, the work done will be path independent.

ELECTROSTATIC POTENTIAL DIFFERENCE

Electrostatic potential difference between two points P and Q of a charge configuration consisting of charges $q_1, q_2\, q_3, q_4$, and q is equal to the work done by an external force in moving a unit positive test charge against the electrostatic force from point Q to P along any path between these two points. Figure shows that work done on a test charge q_0 by the electrostatic field due to any given charge configuration depends only on the position of initial point Q and position of final point P. Work done is independent of the path chosen in going from Q to P.



Electrostatic potential difference between two points P and Q

If V_Q and V_P are the electrostatic potentials at Q and P respectively, then electrostatic potential difference between points Q and P is

$$\Delta V = V_P - V_Q$$

Thus, $$\boxed{\Delta V = \frac{W_{QP}}{q_0}}$$

The dimensional formula for electrostatic potential difference is given by

$$\Delta V = \frac{W_{QP}}{q_0} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{AT}]} = [\text{ML}^2\text{T}^{-3}\text{A}^{-1}]$$

The SI unit of electrostatic potential difference is **volt**.

$$1\text{ V} = 1\text{ JC}^{-1} = 1\text{ N-m C}^{-1}$$

Thus, electrostatic potential difference between any two points in an electrostatic field is said to be **one volt**, when one joule of work is done by an external force in moving a positive charge of one coulomb from one point to the other against the electrostatic force of field without any acceleration.

Note **One electron-volt** (1 eV) is the energy equal to the work required to move a single elementary charge e such as an electron or the proton through a potential difference of exactly one volt (1 V).

$\therefore$ $1\text{ eV} = e\,(1\text{V}) = (1.60\times10^{-19}\text{C})\,(1\text{J/C}) = 1.60\times10^{-19}$ J

EXAMPLE |2| The potential difference between two points is 20 V. How much work will be done in carrying a charge of 400 μC from one point to the another?

Sol. Given, $\Delta V = 20$ V and $q = 400\,\mu\text{C} = 400\times10^{-6}$ C

We know that,

$$\text{Electrostatic potential difference} = \frac{\text{Work done}}{\text{Charge}}$$

$$\Rightarrow \quad \Delta V = \frac{W}{q}$$

$$\Rightarrow \quad 20 = \frac{W}{400\times10^{-6}}$$

$$\therefore \quad W = 20\times400\times10^{-6} = 8\times10^{-3}\text{ J}$$

EXAMPLE |3| If 100 J of work must be done to move an electric charge of magnitude 4 C from a place A, where potential is –10 V to another place B where potential is V volt. Find the value of V.

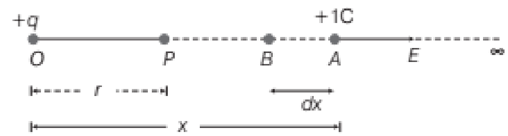
Sol. Given, $W_{AB} = 100$ J, $q = 4$ C, $V_A = -10$ V, $V_B = V = ?$

Since, $W_{AB} = q(V_B - V_A)$

$\Rightarrow \quad 100 = 4(V + 10) \Rightarrow V = 15$ V

ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

Let P be the point at a distance r from the origin O at which the electric potential due to charge $+q$ is required.



The electric potential at a point P is the amount of work done in carrying a unit positive charge from ∞ to P. As, work done is independent of the path, we choose a convenient path along the radial direction from infinity to the point P without acceleration. Let A be an intermediate point on this path where $OA = x$. The electrostatic force on a unit positive charge at A is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times 1}{x^2} \quad \text{[along } OA] \quad \ldots\text{(i)}$$

Small work done in moving the charge through a distance dx from A to B is given by

$$dW = \mathbf{F} \cdot d\mathbf{x}$$
$$= Fdx \cos 180° = -Fdx \quad [\because \cos 180° = -1]$$
$$\Rightarrow \quad dW = -Fdx \quad \ldots\text{(ii)}$$

Total work done in moving a unit positive charge from ∞ to the point P is given by

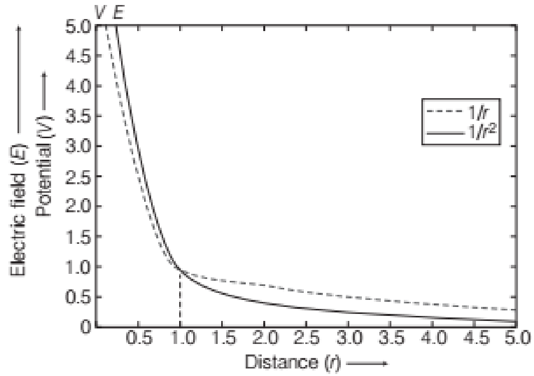
$$W = \int_{\infty}^{r} -F\,dx$$
$$= \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}\,dx$$
$$= -\frac{q}{4\pi\varepsilon_0} \int_{\infty}^{r} x^{-2}\,dx$$
$$= -\frac{q}{4\pi\varepsilon_0} \left[\frac{-1}{x}\right]_{\infty}^{r} \quad \left[\because \int x^{-2}\,dx = -\frac{1}{x}\right]$$
$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r} - \frac{1}{\infty}\right]$$
$$\Rightarrow \quad W = \frac{q}{4\pi\varepsilon_0 r} \quad \ldots\text{(iii)}$$

From the definition of electric potential, this work is equal to the potential at point P.

$$\boxed{V = \frac{q}{4\pi\varepsilon_0 r}} \quad \ldots\text{(iv)}$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential. Here, we assume that electrostatic potential is zero at infinity. Eq.(iv) shows that at equal distances from a point charge q, value of V is same.

Hence, electrostatic potential due to a single charge is spherically symmetric. Figure given below shows the variation of electrostatic potential with distance, i.e. $V \propto \frac{1}{r}$ and also the variation of electrostatic field with distance, i.e. $E \propto \frac{1}{r^2}$.



Variation of electrostatic potential V and electric field E with distance r

Due to a single charge, $F \propto \frac{1}{r^2}$, $E \propto \frac{1}{r^2}$ but $V \propto \frac{1}{r}$, where r is the distance from the charge.

EXAMPLE |4| What is the electrostatic potential at the surface of a silver nucleus of diameter 12.4 fermi? Atomic number (Z) for silver is 47.

Sol. Given, $r = \frac{12.4}{2} = 6.2 \text{ fermi} = 6.2 \times 10^{-15}\text{ m}$ and $Z = 47$

$\therefore$ Charge of the nucleus, $q = Ze = 47 \times 1.6 \times 10^{-19}$ C

$[\because e = 1.6 \times 10^{-19}\text{C}]$

$\therefore$ Electrostatic potential at the surface,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{9 \times 10^9 \times 47 \times 1.6 \times 10^{-19}}{6.2 \times 10^{-15}} = 1.09 \times 10^7\text{V}$$

EXAMPLE |5| A point charge Q is placed at point O as shown in the figure. Is the potential difference $(V_A - V_B)$ positive, negative or zero, if Q is

(i) positive? (ii) negative?

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Sol. Let the distance of points A and B from charge Q be r_A and r_B, respectively.

$\therefore$ Potential difference between points A and B,

$$V_A - V_B = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

As, $r_A = OA$, $r_B = OB$ and $r_A < r_B$

$$\Rightarrow \quad \frac{1}{r_A} > \frac{1}{r_B}$$

Therefore, $\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$ has positive value.

$(V_A - V_B)$ depends on the nature of charge Q.

(i) $(V_A - V_B)$ is positive when $Q > 0$, then

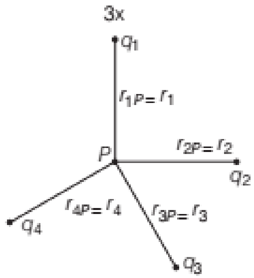
(ii) $(V_A - V_B)$ is negative when $Q < 0$.

ELECTROSTATIC POTENTIAL DUE TO A SYSTEM OF CHARGES

Let there be a number of point charges $q_1, q_2, q_3, \ldots, q_n$ at distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the point P, where electric potential is to be calculated.

Potential at P due to charge q_1,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1P}}$$



A system of charges

Similarly, $V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2P}}$, $V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_{3P}}$, ...,

$$V_n = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_n}{r_{nP}}$$

Using superposition principle, we obtain resultant potential at point P due to total charge configuration as the algebraic sum of the potentials due to individual charges.

$\therefore \quad V = V_1 + V_2 + V_3 + \cdots + V_n$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1P}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2P}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_{3P}} + \cdots + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_n}{r_{nP}}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} + \cdots + \frac{q_n}{r_{nP}}\right)$$

$$\Rightarrow \quad \boxed{V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{iP}}}$$

The net electrostatic potential at a point due to multiple charges is equal to the algebraic sum of the potentials due to individual charges at that particular point.

Mathematically, it is expressed as

$$V_{\text{net}} = \sum_{i=1}^{n} V_i$$

Important Results

- If $r_1, r_2, r_3, \ldots, r_n$ are position vectors of the charges $q_1, q_2, q_3, \ldots, q_n$ respectively, then electrostatic potential at point P whose position vector is r_0, would be

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|r_0 - r_i|}$$

- If we have to calculate electric potential due to a continuous charge distribution characterised by volume charge density $\rho(r)$, we divide the entire volume into a large number of small volume elements each of volume ΔV.

  Charge on each element $= \rho \Delta V$. $\left[\because \rho = \frac{q}{\Delta V}\right]$

- For a uniformly charged conducting spherical shell, the electric field outside the shell is as, if the entire charge is concentrated at the centre. Thus, the potential outside the shell is given by

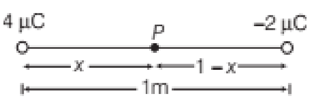
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \qquad [\because r \geq R]$$

where, q is the total charge on the shell and R is its radius. The electric field inside the shell is zero. This implies that potential is constant inside the shell (as no work is done in moving a charge inside the shell) and therefore equal to its value at the surface, which is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

EXAMPLE |6| Two point charges of 4 μC and −2 μC are separated by a distance of 1 m in air. Find the location of a point on the line joining the two charges, where the electric potential is zero.

Sol. Let the electrostatic potential be zero at point P between the two charges separated by a distance x metre.



At point P, $\quad V_P = V_1 + V_2 = 0$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{4 \times 10^{-6}}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-2 \times 10^{-6})}{(1-x)} = 0$$

$$\Rightarrow \quad \frac{4 \times 10^{-6}}{x} = \frac{2 \times 10^{-6}}{(1-x)}$$

$$\Rightarrow \quad \frac{4}{x} = \frac{2}{(1-x)}$$

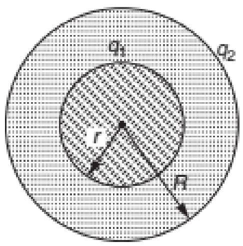
$$\Rightarrow \quad 2(1-x) = x$$

$$\Rightarrow \quad 2 = 3x \quad \text{or} \quad x = \frac{2}{3}$$

$\therefore$ Electrostatic potential is zero at a distance 2/3 m from charge 4μC between the two charges.

EXAMPLE |7| A charge Q is distributed over two concentric hollow spheres of radii r and R ($>r$) such that the surface densities are equal. Find the potential at the common centre.

Sol. Let q_1 and q_2 be the charges on them.



$$\sigma_1 = \sigma_2$$

$$\therefore \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$$\therefore \quad \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

i.e. charge on them is distributed in above ratio

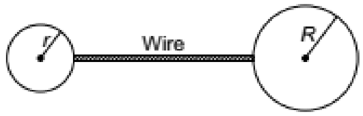
or $q_1 = \dfrac{r^2}{r^2 + R^2} Q$ and $q_2 = \dfrac{R^2}{r^2 + R^2} Q$

$\therefore$ Potential at centre

$V =$ Potential due to q_1 + Potential due to q_2

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{R} = \frac{Q(R+r)}{4\pi\varepsilon_0 (r^2 + R^2)}$$

EXAMPLE |8| Two spherical metal shells with different radii r and R are far apart and connected by a thin conducting wire. A charge Q is placed on one of them. The charge redistributes so that same is on each sphere. How much charge is on the sphere with radius r?



Sol. The electrical potential of a spherical shell with charge q and radius r is kq/r, where $k = 1/(4\pi\varepsilon_0)$.

Since, the shells are joined by a conductor the charge will distribute between them so that they attain the same electrical potential.

Let the charge on the sphere with radius r be q_r and that on the another sphere q_R. Then, equating the potentials gives $q_r / r = q_R / R$.

$$\Rightarrow \quad q_r = q_R (r/R) \quad \text{...(i)}$$

$\because$ The total charge equals the original charge.

$$\therefore \quad Q = q_r + q_R \Rightarrow q_R = Q - q_r$$

By Eq. (i), $\quad q_r = (Q - q_r)(r/R)$

Solving for q_r gives $\quad q_r (1 + r/R) = Q(r/R)$

$$\Rightarrow \quad q_r = Qr/(R + r)$$

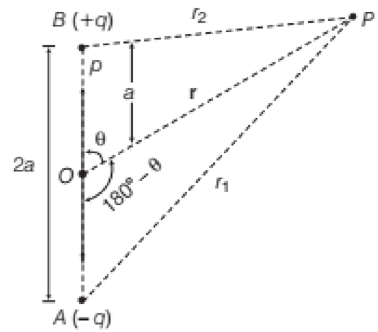
which is the required charge.

Note The value of k is not needed, stating proportionally is sufficient.

ELECTROSTATIC POTENTIAL DUE TO AN ELECTRIC DIPOLE

Let us consider an electric dipole consisting of charges $+q$ and $-q$ separated by a distance $2a$.

The dipole moment $|\mathbf{p}| = q \times 2a$.



Electric potential at point P due to electric dipole

Let O be the centre of the dipole, P be any point near the electric dipole inclined at an angle θ as shown in the figure.

Let P be the point at which electric potential is required.

Potential at P due to $-q$ charge, $V_1 = \dfrac{-q}{4\pi\varepsilon_0 r_1}$

Potential at P due to $+q$ charge, $V_2 = \dfrac{q}{4\pi\varepsilon_0 r_2}$

As, potential is related to work done by the field, electrostatic potential also follows the superposition principle. Therefore, potential at P due to the dipole,

$$V_P = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \quad \text{...(i)}$$

Now, by geometry,

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta$$

Similarly, $\quad r_2^2 = r^2 + a^2 + 2ar\cos(180° - \theta)$

or $\quad r_2^2 = r^2 + a^2 - 2ar\cos\theta \quad [\because \cos(180° - \theta) = -\cos\theta]$

and $\quad r_1^2 = r^2\left(1 + \dfrac{a^2}{r^2} + \dfrac{2a}{r}\cos\theta\right)$

If $r >> a$, $\frac{a}{r}$ is small.

Therefore, $\frac{a^2}{r^2}$ can be neglected.

$$r_1^2 = r^2\left(1+\frac{2a}{r}\cos\theta\right)$$

$$\Rightarrow \quad r_1 = r\left(1+\frac{2a}{r}\cos\theta\right)^{1/2}$$

or
$$\frac{1}{r_1} = \frac{1}{r}\left(1+\frac{2a}{r}\cos\theta\right)^{-1/2}$$

Similarly,
$$\frac{1}{r_2} = \frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-1/2}$$

Putting these values in Eq. (i), we obtain

$$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}} - \frac{1}{r}\left(1+\frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}\right]$$

Using Binomial theorem, $[(1+x)^n = 1+nx, x << 1]$ and retaining terms upto the first order in $\frac{a}{r}$, we set

$$V_P = \frac{q}{4\pi\varepsilon_0 r}\left[\left(1+\frac{a}{r}\cos\theta\right)-\left(1-\frac{a}{r}\cos\theta\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{2a\cos\theta}{r}\right)$$

$$= \frac{q\times 2a\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \quad \boxed{V_P = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}} \qquad [\because p = q\times 2a]$$

As, $\quad p\cos\theta = \mathbf{p}\cdot\hat{\mathbf{r}}$

where, $\hat{\mathbf{r}}$ is a unit vector along the position vector $\mathbf{OP} = \mathbf{r}$.

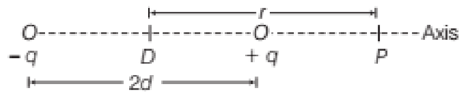
$\therefore$ Electrostatic potential at point P due to a short dipole ($a << r$) is given by $\boxed{V = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}}$

The potential depends just not only on the position vector **r**, but also on the angle between the position vector **r** and the dipole moment **P**. The electric potential due to an electric dipole at point P varies inversely with square of r, i.e. the distance of point P from the centre of the dipole.

Electrostatic Potential due to Dipole on its Axis and Equatorial Plane

On the dipole axis, $\quad \theta = 0°$ or π

$$\therefore \quad V = \pm\frac{p}{4\pi\varepsilon_0 r^2}$$

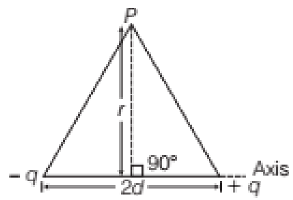


Positive sign for $\theta = 0°$ and negative sign for $\theta = \pi$.

In the equatorial plane, $\theta = \frac{\pi}{2}$

$$\cos\theta = \cos\frac{\pi}{2} = 0$$

$$\therefore \quad V = 0$$



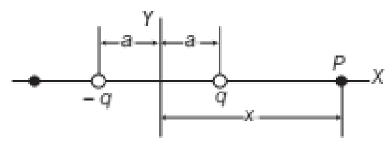
Thus, electrostatic potential at any point in the equatorial plane of dipole is zero.

Differences between electric potential due to an electric dipole and due to a single charge are given as below:

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector **r** and dipole moment vector **p**.

(ii) The electric potential due to dipole falls off at large distance as $1/r^2$ not as $1/r$, which is a characteristic of the potential due to single charge.

EXAMPLE |9| An electric dipole consists of two charges of equal magnitude and opposite signs separated by a distance $2a$ as shown in figure. The dipole is along the X-axis and is centred at the origin.



(i) Calculate the electric potential at point P.

(ii) Calculate V at a point far from the dipole.

Sol. (i) For the point P in figure,

$$V = k_e\,\Sigma\frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_e qa}{x^2 - a^2}$$

(ii) If point P is far from the dipole, such that $x >> a$, then a^2 can be neglected in the terms, $x^2 - a^2$ and V becomes

$$V = \frac{2k_e qa}{x^2} \qquad [\because x >> a]$$

EQUIPOTENTIAL SURFACES

Any surface which has same electrostatic potential at every point, on it is called an equipotential surface. For a single charge q, the potential is given by $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$. This indicates that V is a constant, if r is constant. Thus, the equipotential surface for single point charge are spherical surfaces centred at the charge.The equipotential surfaces can be drawn through any region in which there is electric field. If all the points at same potential in the electric field are joined, then an equipotential surface is obtained.

The shape of equipotential surface due to a

(i) line charge is cylindrical (ii) point charge is spherical.

Different properties of equipotential surfaces are given as below:

(i) Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point which is not possible.

(ii) Equipotential surfaces are closely spaced in the region of strong electric field and widely spaced in the region of weak electric field.

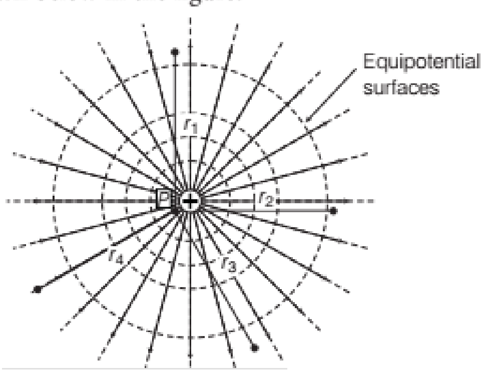
(iii) For any charge configuration, equipotential surface through a point is normal to the electric field at that point and directed from one equipotential surface at higher potential to the other equipotential surface at lower potential.

(iv) No work is required to move a test charge on an equipotential surface.

(v) For a uniform electric field E, let along X-axis, the equipotential surfaces are normal to the X-axis, i.e. planes parallel to the YZ-plane.

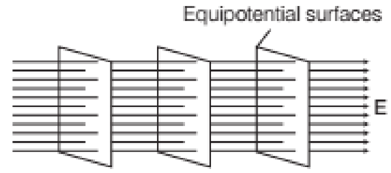
Equipotential Surfaces in Different Cases

Case I The equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution is a family of concentric spheres as shown below in the figure.



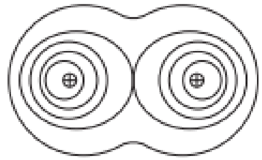
Equipotential surfaces for a point charge

Case II The equipotential surfaces for a uniform electric field are as shown below in figure by dotted lines.



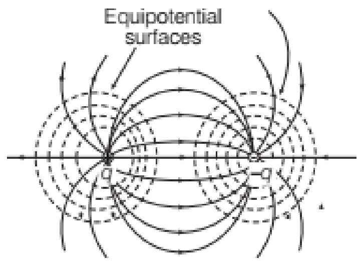
Equipotential surfaces for a uniform electric field

Case III The equipotential surfaces due to two identical positive charges are as shown below



Equipotential surfaces due to two positive charges

Case IV The equipotential surfaces for an electric dipole are as shown below in the figure by dotted lines.



Equipotential surfaces due to an electric dipole

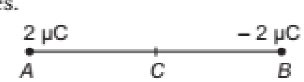
Electric field is always perpendicular to an equipotential surface and as a result, work done in moving a charge between two points on an equipotential surface is zero.

Note This topic has been frequently asked in previous years 2014, 2013, 2011, 2010.

EXAMPLE |10| Two charges $2\mu C$ and $-2\,\mu C$ are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system. **Delhi 2013**

Sol. Equipotential surface means the surface where potential remains same at each point.

Here, this is the system of two equal and opposite charges.

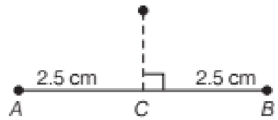


$\therefore$ The potential at C (mid-point of AB),

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2\times10^{-6}}{2.5\times10^{-2}} + \frac{(-2\times10^{-6})}{2.5\times10^{-2}}\right] = 0$$

Thus, potential is zero at each point on the line which passes through the mid-point of AB and perpendicular to it. So, a plane passing through the mid-point C of AB is an equipotential surface.

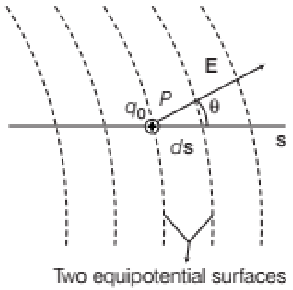


Relation between Electric Field and Electric Potential

Let us consider a positive test charge (q_0) moves a distance (ds) from one equipotential surface to another. The displacement (ds) makes an angle (θ) with the direction of the electric field (**E**).

Suppose a positive test charge (q_0) moves through a differential displacement $d\mathbf{s}$ from one equipotential surface to the adjacent surface.

We know that the work done by the electric field on the test charge during its movement is $-q_0\, dV$. We see that the work done by the electric field may also be written as the scalar product ($q_0\mathbf{E}\cdot d\mathbf{s}$) or $q_0 E\cos\theta\, ds$.



Displacement of charge between two equipotential surfaces

Equating these two expressions for the work yields

$$-q_0 dV = q_0 E\cos\theta\, ds$$

$$\Rightarrow \qquad E\cos\theta = -\frac{dV}{ds}$$

Since, $E\cos\theta$ is the component of **E** in the direction of ds, therefore

$$E_s = -\frac{\partial V}{\partial s}$$

where E_x, E_y and E_z are the x, y and z-components of **E** at any point, then

$$E = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$$

$$\because \qquad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \qquad E = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

For the simple situation in which the electric field **E** is uniform.

$$E = -\frac{\Delta V}{\Delta s}$$

Negative sign shows that the direction of electric field **E** in the direction of decreasing potential.

Since, ΔV is negative, then $\Delta V = -|\Delta V|$

We can rewrite this equation as given below

$$|E| = -\frac{\Delta V}{\Delta s} = +\frac{|\Delta V|}{\Delta s}$$

Further, the magnitude of an electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point. This is called **potential gradient**, i.e.

$$|\mathbf{E}| = -\frac{|d\mathrm{V}|}{ds} = -\text{(Potential gradient)}$$

We thus arrive at two important conclusions concerning the relation between electric field and potential which are as given below

(i) Electric field is in the direction in which the potential decreases steepest.

(ii) Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

EXAMPLE |11| A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between the horizontal metal plates 5 cm apart, having a potential difference of 3000 V across them. Find the mass of the particle.

Sol. Here, $q = -1.6\times10^{-19}$ C,

$dr = 5\text{ cm} = 5\times10^{-2}$ m

and $dV = 3000$ V

$$\therefore \qquad E = -\frac{\partial V}{\partial r} = \frac{-3000}{5\times10^{-2}} = -6\times10^4\ \text{Vm}^{-1}$$

As, the charged particle remains suspended in equilibrium, therefore

$$F = mg = qE$$

$$\therefore \quad m = \frac{qE}{g} = \frac{(-1.6\times10^{-19})\times(-6\times10^{4})}{9.8}$$

$$= 9.8\times10^{-16}\text{ kg}$$

EXAMPLE |12| The electric potential in a region is represented as

$$V = 2x + 3y - z.$$

Obtain expression for electric field strength.

Sol. As, $\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$

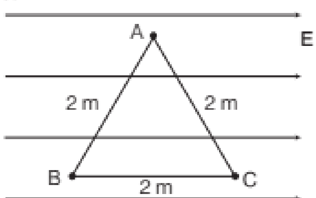
Here, $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + 3y - z) = 2$

$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$

and $\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x + 3y - z) = -1$

$\therefore$ Electric field, $\mathbf{E} = -2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

EXAMPLE |13| In uniform electric field, $\mathbf{E} = 10\text{ NC}^{-1}$ as shown in figure.



Find

(i) $V_A - V_B$ (ii) $V_B - V_C$

Sol. Since, electric field is directed from higher electric potential to lower electric potential,

(i) Thus, $V_B > V_A$, so $V_A - V_B$ will be negative.
Further, $d_{AB} = 2\cos 60° = 1$ m
$\therefore \quad V_A - V_B = -Ed_{AB} = (-10)(1) = -10$ V

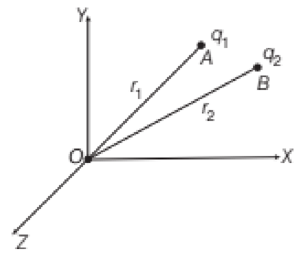
(ii) As, $V_B > V_C$, so $V_B - V_C$ will be positive.
Further, $d_{BC} = 2.0$ m
$\therefore \quad V_B - V_C = (10)(2) = 20$ V

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.

Electrostatic Potential Energy of a System of Two Point Charges

Consider two point charges q_1 and q_2 lying at points A and B whose locations are $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. To find the electric potential energy of these two charges system, we must mentally build the system starting with both charges infinitely far away and at rest. First, the charge q_1 is brought from infinity to the point r_1. There is no external field against which work needs to be done, so work done in bringing q_1 from infinity to r_1 is zero. V is potential that has been set up by q_1 at the point B, where q_2 is to be placed.



$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_{AB}}$$

where, r_{AB} is the distance between points A and B.

By definition, work done in carrying charge q_2 from ∞ to B is

$$W = \text{Potential} \times \text{Charge} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_{AB}}\cdot q_2$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{AB}}$$

This work is stored in the system of two point charges q_1 and q_2 in the form of electrostatic potential energy U of the system.

Thus, $$U = W = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{AB}}$$

Electrostatic potential energy is a scalar quantity. In the above formula, the values of q_1 and q_2 must be with proper signs. If $q_1, q_2 > 0$, then potential energy is positive. It means that two charges are of same sign, i.e. they repel each other. Then, in bringing closer, work is done against the force of repulsion, so that the electrostatic potential energy of the system increases.

Conversely, in separating them, work is obtained from the system, so the potential energy of the system decreases.

If $q_1 > 0, q_2 < 0$, potential energy is negative. It means that two charges are of opposite sign, i.e. they attract each other. In this case, potential energy of the system decreases in bringing them closer and increases in separating them further.

EXAMPLE |14| In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å.

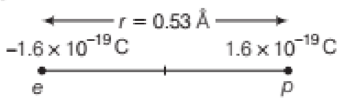
(i) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(ii) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (i)?

(iii) What are the answers to (i) and (ii) above, if the zero of potential energy is taken at 1.06 Å separation? **NCERT**

> **Hints:** The potential energy of any object at any point is equal to the difference in its potential energy at infinity and at that point. Work done is equal to the total energy of the system.

Sol. Charge on electron, $q_e = -1.6 \times 10^{-19}$ C and charge on proton, $q_p = 1.6 \times 10^{-19}$ C



(i) Potential energy of the system

= Potential energy at infinity

− Potential energy at a distance of 0.53 Å

$$= 0 - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_e q_p}{r}$$

$$= 0 - \frac{9 \times 10^9 \times (-1.6) \times 10^{-19} \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}}$$

$$= -43.47 \times 10^{-19} \text{ J} \qquad [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$= -\frac{43.47 \times 10^{-19}}{1.6 \times 10^{-19}} = -27.16 \text{ eV}$$

(ii) The kinetic energy $= -\frac{1}{2} \times$ Potential energy

$$= -\frac{1}{2} \times (-27.16) = 13.58 \text{ eV}$$

Total energy = KE + PE = 13.58 − 27.16 = −13.58 eV

Thus, the minimum work done required to free the electron is 13.58 eV.

(iii) Potential energy at separation of 1.06 Å

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_e q_p}{1.06 \times 10^{-10}}$$

$$= \frac{-9 \times 10^9 \times (-1.6) \times 10^{-19} \times 1.6 \times 10^{-19}}{1.06 \times 10^{-10}}$$

$$= -21.73 \times 10^{-19} \text{ J}$$

$$= -\frac{21.73 \times 10^{-19}}{1.6 \times 10^{-19}} = -13.58 \text{ eV}$$

Thus, the potential energy of the system at 1.06 Å

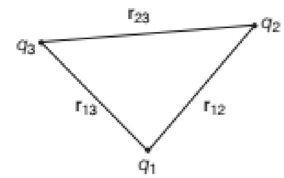
= PE at distance 1.06 Å − PE at distance 0.53 Å

= − 13.58 − (− 27.16) = 13.58 eV

Thus, on shifting the zero of potential energy, work required to free electron remains same and it is equal to 13.58 eV.

Electrostatic Potential Energy of a System of Three Point Charges

Let us now consider a system of three point charges q_1, q_2 and q_3 having position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$, respectively as from origin.



Three point charges system

To bring q_1 first from infinity to position r_1, no work is required because we bring charge q_1 from infinity to a particular location where potential is zero.

Therefore, $W_1 = 0$

The work done in bringing q_2 from infinity to position r_2 is given by $W_2 = q_2 V_1(r_2)$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

Charges q_1 and q_2 produce a potential which at any point P is given by

$$V_{1,2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

Work done in bringing q_3 from infinity to position r_3 is q_3 times $V_{1,2}$ at r_3,

$$W_3 = q_3 V_{1,2}(r_3)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

The total work done in assembling the charges at the given locations (equal to the potential energy of the system) is obtained by adding the work done in different steps.

$$U = W_1 + W_2 + W_3$$

$$\Rightarrow \quad U = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\Rightarrow \quad \boxed{U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)}$$

This result can also be expressed in summation form as

$$\boxed{U = \left[\frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{3} \sum_{\substack{j=1 \\ j \neq i}}^{3} \frac{q_i q_j}{r_{ij}}\right]}$$

Due to the conservative nature of electrostatic force, the value of U is independent of the manner in which the configuration is assembled.

If we write the distance $|\mathbf{r}_i - \mathbf{r}_j|$ as r_{ij}, the above equation may be expressed as for system of n point charges system.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{q_i q_j}{r_{ij}}\right]$$

Electrostatic potential energy of a system of N point charges is equal to the total amount of work done in assembling all the charges at the given positions from infinity.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j \neq 1}}^{N} q_i V_j\right]$$

where, $V_j = \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_j}{r_{ij}}$

= Potential at $\mathbf{r}_j$ due to all other charges

The SI unit of electrostatic potential energy is joule (J). Another convenient unit of energy is electron volt (eV).

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$$

EXAMPLE |15| Three charges (all $q = 10\text{C}$) are placed at the edge of an equilateral triangle of side 2 m. Find the net potential energy of the system.

Sol. Given, charge, $q = 10\text{C}$ $(q_1 = q_2 = q)$

Each side of equilateral triangle, $r = 2\text{m}$

Potential energy (PE) = ?

Potential energy between two charges is given by

$$\text{PE} = \frac{kq_1q_2}{r} \quad [\because r = \text{distance between } q_1 \text{ and } q_2]$$

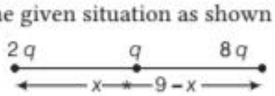
$\therefore$ PE of system will be three times the potential energy between the two charges as the equal charge is placed at the vertices of equilateral triangle.

$$\text{So, PE}_{\text{net}} = \frac{3 \times kqq}{r} = \frac{3\,kq^2}{r} = \frac{3 \times 9 \times 10^9 \times 10 \times 10}{2}$$

$$= 135 \times 10^{10} \text{ J}$$

EXAMPLE |16| Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges ?

Sol. Consider the given situation as shown in figure.



For potential energy to be minimum the bigger charges should be farthest. Let x be the distance of q from $2q$. Then potential energy of the system shown in figure would be

$$U = K\left[\frac{(2q)(q)}{x} + \frac{(8q)(q)}{(9-x)} + \frac{(2q)(8q)}{9}\right]$$

Here, $K = \frac{1}{4\pi\varepsilon_0}$

For U to be minimum $\frac{2}{x} + \frac{8}{9-x}$ should be minimum.

$$\frac{d}{dx}\left[\frac{2}{x} + \frac{8}{9-x}\right] = 0$$

$$\Rightarrow \quad \frac{-2}{x^2} + \frac{8}{(9-x)^2} = 0$$

$$\Rightarrow \quad \frac{x}{9-x} = \frac{1}{2}$$

or $\quad x = 3$ cm

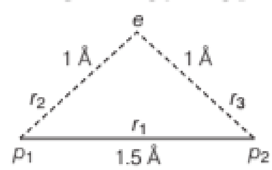
i.e. distance of charge q from $2q$ should be 3 cm.

$\therefore$ Electric field at q,

$$E = \frac{K(2q)}{(3 \times 10^{-2})^2} - \frac{K(8q)}{(6 \times 10^{-2})^2} = 0$$

EXAMPLE |17| If one of the two electrons of H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy. **NCERT**

Sol. Let there are two protons p_1 and p_2 with an electron e.



Distance between two protons is given by

$$r_1 = 1.5 \text{ Å} = 1.5 \times 10^{-10} \text{ m}$$

Distance between proton p_1 and electron e is given by

$$r_2 = 1 \text{ Å} = 1 \times 10^{-10} \text{ m}$$

Distance between proton p_2 and electron e is given by

$$r_3 = 1 \text{ Å} = 1 \times 10^{-10} \text{ m}$$

The total potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q_{p_1}q_{p_2}}{r_1} + \frac{q_{p_1}q_e}{r_2} + \frac{q_{p_2}q_e}{r_3}\right] \quad \text{...(i)}$$

Given, $q_{p_1} = q_{p_2} = 1.6 \times 10^{-19}\text{C}$

and $\quad q_e = -1.6 \times 10^{-19}\text{C}$

Putting these values in Eq. (i), we get

$$U = 9\times 10^9\left[\frac{1.6\times 10^{-19}\times 1.6\times 10^{-19}}{1.5\times 10^{-10}} + \frac{(1.6\times 10^{-19})\times(-1.6\times 10^{-19})}{10^{-10}} + \frac{1.6\times 10^{-19}\times(-1.6\times 10^{-19})}{10^{-10}}\right]$$

$$= \frac{9\times 10^9\times 1.6\times 1.6\times 10^{-38}}{10^{-10}}\left[\frac{1}{1.5}-1-1\right]$$

$$= -30.72\times 10^{-19}\ \text{J}$$

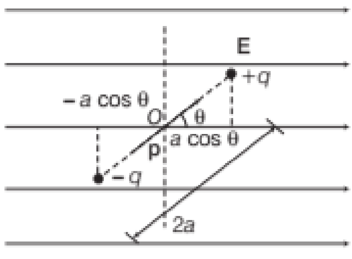
$$= \frac{-30.72\times 10^{-19}}{1.6\times 10^{-19}}\ \text{eV} = -19.2\ \text{eV}$$

Here, we use that potential energy at infinity is zero.

POTENTIAL ENERGY IN AN EXTERNAL FIELD

A single charge or a system of charges possess electrostatic potential energy in the presence of an external electric field, these are discussed as follows.

applied to the dipole so that it rotates from angle θ_1 to θ_2 with respect to the electric field (E).



Dipole in a uniform external field

The amount of work done by the external torque is given by

$$W = \int_{\theta_1}^{\theta_2} \tau_{ext}\,\theta\, d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$= pE\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= pE\,(\cos\theta_1 - \cos\theta_2)$$

The work done W is stored as the potential energy of the system. Therefore, the potential energy of the dipole placed in external field **E** is given by

$$\boxed{U(\theta) = pE\,(\cos\theta_1 - \cos\theta_2)}$$

POTENTIAL ENERGY IN AN EXTERNAL FIELD

A single charge or a system of charges possess electrostatic potential energy in the presence of an external electric field, these are discussed as follows.

Potential Energy of a Single Charge in External Field

Potential energy of a single charge q at a point with position vector $\mathbf{r}$ in an external field $= q \cdot V(\mathbf{r})$, where $V(\mathbf{r})$ is the potential at the point due to external electric field $\mathbf{E}$.

Potential Energy of a System of Two Charges in an External Field

For a system of two charges q_1 and q_2, the potential energy is given as,

$$U = q_1 \cdot V(\mathbf{r}_1) + q_2 \cdot V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

where, $q_1, q_2 =$ two point charges at position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively

$V(\mathbf{r}_1) =$ potential at $\mathbf{r}_1$ due to the external field

and $V(\mathbf{r}_2) =$ potential at $\mathbf{r}_2$ due to the external field.

Potential Energy of a Dipole in an External Field

Consider a dipole with charges $+q$ and $-q$ placed in a uniform external electric field as shown in the figure. In a uniform electric field, the dipole experiences no force, but experiences a torque τ given by $\tau = \mathbf{p} \times \mathbf{E}$.This torque will tend to rotate the dipole. Suppose an external torque τ_{ext} is

$$= pE\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= pE\,(\cos\theta_1 - \cos\theta_2)$$

The work done W is stored as the potential energy of the system. Therefore, the potential energy of the dipole placed in external field $\mathbf{E}$ is given by

$$U(\theta) = pE\,(\cos\theta_1 - \cos\theta_2)$$

Particular Cases

(i) When the dipole is initially aligned along the electric field, i.e. $\theta_1 = 0°$ and we have to set it at angle θ with $\mathbf{E}$, i.e. $\theta_2 = \theta$.

$\therefore \quad W = -pE(\cos\theta - \cos 0°)$

$= -pE(\cos\theta - 1)$

This work done is stored in the dipole in the form of potential energy.

(ii) When the dipole is initially at right angle to $\mathbf{E}$, i.e. $\theta_1 = 90°$ and we have to set it at angle θ with $\mathbf{E}$, i.e. $\theta_2 = \theta$.

$\therefore \quad W = -pE(\cos\theta - \cos 90°)$

$= -pE\cos\theta$

$\therefore$ Potential energy of dipole, $U = W = -pE\cos\theta$

$U = -\mathbf{p} \cdot \mathbf{E}$

Obviously, potential energy of an electric dipole is a scalar quantity. It is measured in joule.

Important Results

Some important results related to electric dipole are as given below

(i) Electric potential at any point on the bisector of dipole is zero.

(ii) A dipole experiences a net force in a non-uniform electric field.

(iii) A dipole experiences maximum torque at the position where potential energy is zero.

EXAMPLE |18| An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ N-m. Calculate the potential energy of the dipole, if it has charge ± 8 nC.

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Sol. Given, length, $2a = 4\text{ cm} = 4\times 10^{-2}$ m

Angle, $\theta = 60°$

torque $\tau = 4\sqrt{3}$ N-m

Charge, $Q = 8\times 10^{-9}$ C

We know that, $\tau = Q(2a)E\sin\theta$

$\Rightarrow$ Electric field, $E = \dfrac{\tau}{Q(2a)\sin\theta}$

$$= \frac{4\sqrt{3}}{8\times 10^{-9}\times 4\times 10^{-2}\times \sin 60°}\text{ N/C}$$

$\therefore$ Potential energy, $U = -pE\cos\theta$

$$= -Q(2a)E\cos\theta$$

$$= -8\times 10^{-9}\times 4\times 10^{-2}\times\left[\frac{4\sqrt{3}\times\cos 60°}{8\times 10^{-9}\times 4\times 10^{-2}\times \sin 60°}\right]$$

$$= \frac{-4\sqrt{3}}{\sqrt{3}} = -4\text{ J}$$

EXAMPLE |19| A point charge q is fixed at origin. A dipole with a dipole moment $\mathbf{p}$ is placed along the X-axis far away from the origin with $\mathbf{p}$ pointing along positive X-axis. Find

(i) the kinetic energy of the dipole when it reaches a distance d from the origin.

(ii) the force experienced by the charge q at this moment.

Delhi 2003

Sol. (i) Applying energy conservation principle, increase in kinetic energy of the dipole = decrease in electrostatic potential energy of the dipole.

$\therefore$ Kinetic energy of dipole at distance d from origin

$$= U_i - U_f = 0 - (-\mathbf{p}\cdot\mathbf{E}) = \mathbf{p}\cdot\mathbf{E}$$

$$= (p\hat{\mathbf{i}})\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\hat{\mathbf{i}}\right) = \frac{qp}{4\pi\varepsilon_0 d^2}$$

(ii) Electric field at origin due to the dipole,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{\mathbf{i}} \qquad [\because \mathbf{E}_{axis}\uparrow\uparrow\mathbf{p}]$$

$\therefore$ Force on charge q,

$$\mathbf{F} = q\mathbf{E} = \frac{pq}{2\pi\varepsilon_0 d^3}\hat{\mathbf{i}}$$

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Which of the following is not a unit of electrostatic potential?

(a) Volt

(b) Joule/coulomb

(c) Newton / Coulomb

(d) Newton - metre / Coulomb

2. Work done by an external force in bringing a unit positive charge from infinity to a point is

(a) equal to the electrostatic potential (V) at that point

(b) equal to the negative of work done by electrostatic forces

(c) Both (a) and (b)

(d) Neither (a) nor (b)

3. To find the value of potential at a point, the external force at every point of the path is to be equal and opposite to the

(a) work done

(b) electrostatic force on the test charge at that point

(c) Both (a) and (b)

(d) Neither (a) nor (b)

4. If electrostatic potential at the surface of a sphere of 5 cm radius is 50 V, then the potential at the centre of sphere will be

(a) 10 V (b) 50 V

(c) 250 V (d) zero

5. The electrostatic potential of a uniformly charged thin spherical shell of charge Q and radius R at a distance r from the centre is

(a) $\dfrac{Q}{4\pi\varepsilon_0 r}$ for points outside and $\dfrac{Q}{4\pi\varepsilon_0 R}$ for points inside the shell

(b) $\dfrac{Q}{4\pi\varepsilon_0 r}$ for both points inside and outside the shell

(c) zero for points outside and $\dfrac{Q}{4\pi\varepsilon_0 r}$ for points inside the shell

(d) zero for both points inside and outside the shell

6. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge **NCERT Exemplar**

(a) remains a constant because the electric field is uniform

(b) increases because the charge moves along the electric field

(c) decreases because the charge moves along the electric field

(d) decreases because the charge moves opposite to the electric field

7. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B. **NCERT Exemplar**

(a) The work done in Fig. (i) is the greatest

(b) The work done in Fig. (ii) is least

(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)

(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

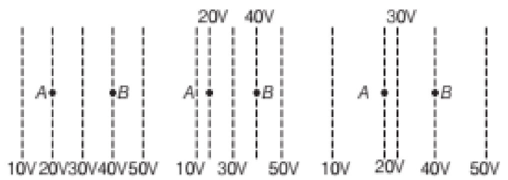


Fig. (i) Fig. (ii) Fig. (iii)

8. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

(a) spheres (b) planes

(c) paraboloids (d) ellipsoids

9. Two similar positive point charges each of $1\,\mu C$ have been kept in air at 1m distance from each other. What will be the potential energy?

(a) 1 J (b) 1 eV

(c) 9×10^{-3} J (d) 0

VERY SHORT ANSWER Type Questions

10. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $(V_A - V_B)$ positive, negative or zero?

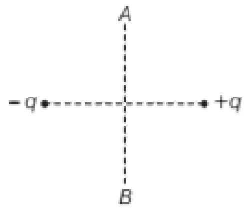
Delhi 2016, Foreign 2016, Delhi 2011

11. Is electrostatic potential necessarily zero at a point, where electric field strength is zero? Illustrate your answer. **Delhi 2010**

12. The potential due to a dipole at any point on its axial line is zero. Correct or Wrong?

All India 2009C

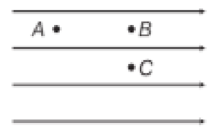
13. A charge q is moved from a point A above a dipole of dipole moment p to a point B below the dipole in equatorial plane without acceleration. Find the work done in this process. **All India 2016**



14. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor? **All India 2016, 2015C**

15. Define the term potential energy for charge q at a distance r in an external field. **All India 2009**

16. For a uniform electric field given as shown below, at what point will the electric potential be maximum?



SHORT ANSWER Type Questions

17. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q.

18. What is the geometrical shape of equipotential surface due to a single isolated charge? **Delhi 2013**

19. Can two equipotential surface intersect each other? Justify your answer. **Delhi 2011**

20. Give the equipotential surface at a great distance from a collection of charges whose total sum is not zero.

21. Two point charges $3\,\mu C$ and $-3\,\mu C$ are placed at points A and B, 5 cm apart.

(i) Draw the equipotential surfaces of the system.

(ii) Why do equipotential surfaces get close to each other near the point charge?

All India 2011

22. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the XZ-plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates. What is the magnitude and direction of this field? **Delhi 2011**

23. Find out the expression for the potential energy of a system of three charges q_1, q_2 and q_3 located at r_1, r_2 and r_3 with respect to the common origin O. **Delhi 2010**

24. Two point charges q_1 and q_2 are located at r_1 and r_2, respectively in an external electric field **E**. Obtain the expression for the total work done in assembling this configuration. **Delhi 2014C**

25. A dipole with its charges, $-q$ and $+q$, located at the points $(0, -b, 0)$ and $(0, +b, 0)$ is present in a uniform electric field **E**. The equipotential surfaces of this field are planes parallel to the YZ- planes.
   (i) What is the direction of the electric field **E**?
   (ii) How much torque would the dipole experience in this field? **Delhi 2010**

26. If a point charge $+q$ is taken from A to C and then from C to B, points A and B lying on a circle drawn with another charge $+q$ at its centre, then along which path more work will be done?

27. Do free electrons travel to region of higher potential or lower potential? **NCERT Exemplar**

28. Prove that a closed equipotential surface with no charge within itself, must enclose an equipotential value.

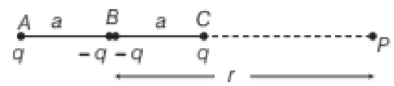
LONG ANSWER Type I Questions

29. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube. **NCERT**

30. (i) Derive the expression for the electric potential due to an electric dipole at a point on its axial line.
   (ii) Depict the equipotential surfaces due to an electric dipole. **Delhi 2017**

31. Give the simplified expression for the following and draw the graph for variation of potential with distance.
   (i) Electrostatic potential due to a point charge q at a distance r from it.
   (ii) General expression for electric potential due to a dipole.

32. Given figure shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$ and contrast your results with that due to an electric dipole and an electric monopole (i.e. a single charge).

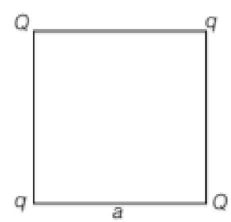


NCERT

33. Define an equipotential surface. Draw equipotential surfaces
   (i) in case of a single point charge
   (ii) in a constant electric field in Z-direction. Why the equipotential surfaces about a single charge are not equidistant?
   (iii) Can electric field exist tangential to an equipotential surface? Give reason.
   All India 2016

34. Three charges $-q$, $+Q$ and $-q$ are placed at equal distance on straight line. If the potential energy of the system of the three charges is zero, then what is the ratio of $Q : q$?

35. Four point charges Q, q, Q and q are placed at the corners of a square of side a as shown in figure.

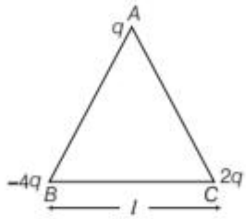


Find the
   (a) resultant electric force on a charge Q and
   (b) potential energy of this system. **CBSE 2018**

Or

   (a) Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side l as shown in the figure.

Obtain the expression for the magnitude of the resultant electric force acting on the charge q.



(b) Find out the amount of the work done to separate the charges at infinite distance. **CBSE 2018**

LONG ANSWER Type II Questions

36. Three concentric metal shells A, B and C of radius a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$, respectively.

(i) Find the potential of three shells at A, B and C.

(ii) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.

37. Two metal spheres, one of radius R and the other of radius $2R$, both have same surface charge density σ. They are brought in contact and separated. What will be new surface charge densities on them? **NCERT Exemplar**

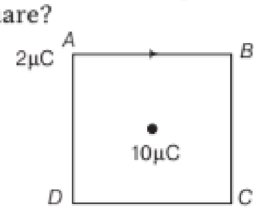
38. (a) Use Gauss' law to derive the expression for the electric field (**E**) due to a straight uniformly charged infinite line of charge density λ C/m.

(b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.

(c) Find the work done in bringing a charge q from perpendicular distance r_1 to $r_2 (r_2 > r_1)$. **CBSE 2018**

NUMERICAL PROBLEMS

39. What is the work done in moving a 2 μC point charge from corner A to corner B of a square $ABCD$, when a 10 μC charge exists at the centre of the square?



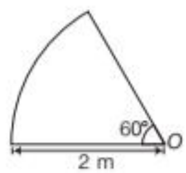
40. The electric potential at 0.1 m from a point charge is + 50 V. What is the magnitude and sign of the charge? **All India 2011**

41. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point (s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. **NCERT**

42. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon. **NCERT**

43. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3) (in cm) to a point Q (0, 4, 0) (in cm), *via* a point R (0, 6, 9) (in cm). **NCERT**

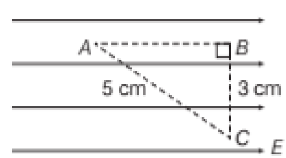
44. The circular arc is shown in the figure given below, has a uniform charge per unit length of 1×10^{-8} C/m. Find the potential at the centre O of the arc.



45. A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between the horizontal metal plates 10 cm apart, having a potential difference of 4000 V across them, find the mass of the particle.

46. An infinite plane sheet of charge density 10^{-8} C/m^2 is held in air. In this situation, how far apart are two equipotential surfaces whose potential difference is 5 V?

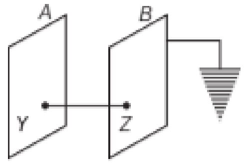
47. A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure.



(i) Calculate the potential difference between A and C.

(ii) At which point (of the two) is the electric potential more and why? **All India 2012**

48. Two identical plane metallic surfaces A and B are kept parallel to each other in air, separated by a distance of 1 cm, surface A is given a positive potential of 10 V and the outer surface of B is earthed.



(i) What is the magnitude and direction of the electric field between the points Y and Z?

(ii) What is the work done in moving a charge of 20 μC from point Y to point Z ?

HINTS AND SOLUTIONS

1. (c) From definition of potential,

$$V = \frac{W}{q} = \frac{F \cdot d}{q} \text{ volt}$$

Here, unit of force is newton, unit of distance (d) is metre and unit of chage (q) is coulomb.

Unit of potential is $\frac{\text{Joule}}{\text{Coulomb}}$ or $\frac{\text{N-m}}{\text{C}}$.

2. (a) Considering potential to be zero at infinity. Work done by an external force in bringing a unit positive charge from infinity to a point without acceleration = Electrostatic potential (V) at that point

3. (b) The external force at every point of the path is to be equal and opposite to the electrostatic force on the test charge at that point.

4. (b) Potential inside a conductor is same at all the points and is equal to the potential at its surface. So, potential at the centre of sphere will also be 50 V.

5. (a) If charge on a conducting sphere of radius R is Q, then potential outside the sphere.

$$V_{\text{out}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

At the surface of sphere,

$$V_s = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = V_{\text{inside}}$$

6. (c) The positively charged particle experiences electrostatic force along the direction of electric field *i.e.*, from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

7. (c) The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

8. (a) In this problem, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = k_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

9. (c) Electric potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

Here, $q_1 = q_2 = 1\,\mu\text{C}$

$= 1 \times 10^{-6}$ C,

$r = 1$ m and $\frac{1}{4\pi\varepsilon_0}$

$= 9 \times 10^9$ N-m^2/C^2

$$\therefore \quad U = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1}$$

$$= 9 \times 10^{-3} \text{ J}$$

10. According to question, $r_2 > r_1$

Potential at point A due to charge $+Q$, $(V_A) = \frac{kQ}{r_1}$

Potential at point B due to charge $+Q$, $(V_B) = \frac{kQ}{r_2}$

As $\quad V_A \propto \frac{1}{r_1}$

and $\quad V_B \propto \frac{1}{r_2}$ and $r_2 > r_1$

so, $\quad V_A > V_B$

Thus, $(V_A - V_B)$ is positive.

11. No, it is not necessary because electric field strength inside a hollow charged spherical shell is zero but potential at the point is same as that on the surface of shell.

12. Wrong, the potential due to a dipole at any point on equatorial line is zero, not on axial line.

13. As, A and B are points on the equitorial plane of dipole $V_A = V_B = 0$

Net potential $= V_A + V_B = 0$

Work done $W = \frac{V}{q}$. As $V = 0$, $W = 0$

So, the work done by the process will be zero.

14. Electric field is always normal to the equipotential surface at every point, because no work is done, as

$$W = q_0(V_A - V_B)$$

$\Rightarrow$ $$V_A - V_B = 0$$

hence $$W = 0.$$

If the field were not normal to the equipotential surface, it would have a non-zero component along the surface. So, to move a test charge against this component, a work would have to be done.

15. The electric potential energy at any point lying at a distance r from the source charge q is equal to the amount of work done in moving unit positive test charge from infinity to that point without any acceleration against electrostatic force.

16. Potential is maximum at A as potential decreases in the direction of field or we can say that $V_A > V_B = V_C$.

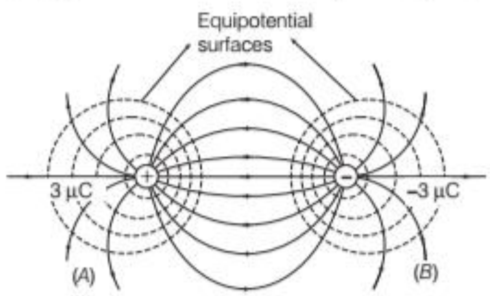
17. Refer to graph on page 64.

18. Refer to text on page 68 (case I).

19. Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point which is not possible.

20. As, the collection of charges at a great distance, so it has spherical equipotential surface.

21. (i) Equipotential surfaces of the system (dipole),



(ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

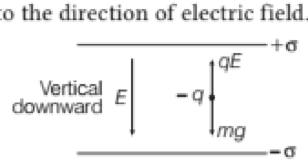
$\because$ $$E = -\frac{\Delta V}{\Delta r}$$

$\Rightarrow$ $$E \propto \frac{1}{\Delta r}$$

[for a given equipotential surface]

where, small Δr represents strong electric field and *vice-versa*.

22. Here, $-q$ charge experiences force in a direction opposite to the direction of electric field.



$\therefore$ $-q$ charge balances, when

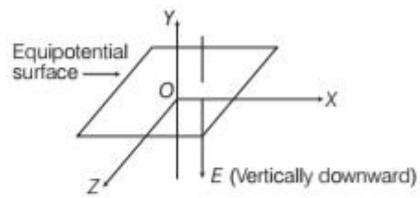
$$qE = mg$$

$\Rightarrow$ $$E = \frac{mg}{q}$$

The direction of electric field is along vertically downward direction.

Note The XZ-plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight (mg) of charged particle could not be balanced.

The sketch of equipotential surface due to electric field between the plates is shown in figure below.



23. Refer to text on page 71.

24. Refer to text on page 73.

25. (i) The direction of electric field is perpendicular to their equipotential surface. So, the direction of electric field is along X-axis as its length should be perpendicular to equipotential surface lying in YZ-plane.

(ii) Length of the dipole = $2b$

As dipole's axis is along the Y-axis.

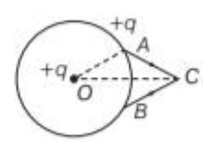
$\therefore$ Electric dipole moment, $\mathbf{p} = q(2b)\hat{\mathbf{j}}$

and electric field, $\mathbf{E} = E\,\hat{\mathbf{i}}$

$\because$ $$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} = q(2b)\hat{\mathbf{j}} \times E\,\hat{\mathbf{i}}$$
$$= +\,2qbE\,(\hat{\mathbf{j}} \times \hat{\mathbf{i}})$$
$$= 2qbE\,(-\hat{\mathbf{k}})$$

$\therefore$ Torque, $|\boldsymbol{\tau}| = 2qbE$

26. Consider the situation as shown in figure.



Work done for the path AC

$$W_{AC} = +q(V_C - V_A)$$

Similarly, $W_{CB} = +q(V_B + V_C)$

$\because$ $$V_A = V_B$$

$\therefore$ $$|W_{AC}| = |W_{CB}|$$

27. The free electrons experience electrostatic force in a direction opposite to the direction of electric field, being of negative charge. The electric field is always directed from higher potential to lower potential.

Therefore, electrostatic force and hence, direction of travelling of electrons is from lower potential to the region of higher potential.

28. **Hints:** In this problem, we need to know that the electric field intensity E and electric potential V are related as $E = -\frac{dV}{dr}$ and the field lines are always perpendicular from one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at a low electrostatic potential.

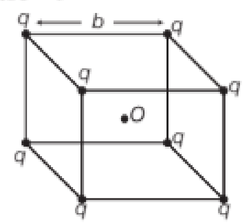
Let's assume contradicting statement that the potential is not same inside the closed equipotential surface. Let the potential just inside the surface be different to that on the surface having a potential gradient $\left(\frac{dV}{dr}\right)$.

Consequently, electric field comes into existence, which is given by, $E = -\frac{dV}{dr}$.

Consequently, field lines point inwards or outwards from the surface. These lines cannot be formed on the surface, as the surface is equipotential. It is possible only when the other end of the field lines are originated from the charges inside. This contradicts the original assumption. Hence, the entire volume inside must be equipotential.

29. Consider a cube of side b and its centre be O. The charge q is placed at each of the corners.

Side of the cube = b



Length of the main diagonal of the cube

$$= \sqrt{b^2 + b^2 + b^2} = \sqrt{3}\,b$$

Distance of centre O from each of the vertices is

$$r = \frac{b\sqrt{3}}{2} \qquad \text{...(i)}$$

Potential at point O due to one charge, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

Potential at point O due to all charges placed at the vertices of the cube,

$$V' = 8V = \frac{8 \times 1 \times q}{4\pi\varepsilon_0 r} = \frac{8q \times 2}{4\pi\varepsilon_0 \cdot b\sqrt{3}} \quad \text{[from Eq. (i)]}$$

$$= \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

The electric field due to one vertex is balanced by the electric field due to the opposite vertex because all charges are positive in nature. Thus, the resultant electric field at the centre O of the cube is zero.

30. (i) Refer to text on page 67.

(ii) Refer to text on page 68 (Case IV).

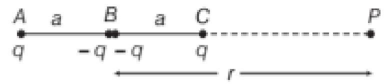
31. Refer text on page 64 for the graph.

(i) Refer to text on pages 63 and 64.

(ii) Refer to text on pages 66 and 67.

32. Given, $AC = 2a$, $BP = r$

$AP = r + a$ and $PC = r - a$



The potential at P is V.

$\therefore$ V = Potential at P due to A + Potential at P due to B + Potential at P due to C

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AP} - \frac{2q}{BP} + \frac{q}{CP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{1}{(r+a)} - \frac{2}{r} + \frac{1}{(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right]$$

$$= \frac{q \cdot 2a^2}{4\pi\varepsilon_0 r(r^2 - a^2)} = \frac{q \cdot 2a^2}{4\pi\varepsilon_0 \cdot r \cdot r^2\left(1 - \frac{a^2}{r^2}\right)}$$

According to the question,

If $\frac{r}{a} >> 1$, $a << r$. Therefore, $V = \frac{q \cdot 2a^2}{4\pi\varepsilon_0 \cdot r^3}$

$$V \propto \frac{1}{r^3}$$

As, we know that electric potential at a point on axial line due to an electric dipole is

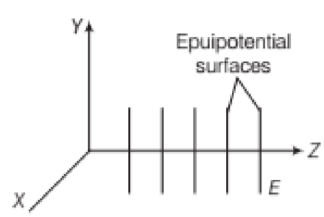
$$V \propto \frac{1}{r^2}$$

In case of electric monopole, $V \propto \frac{1}{r}$.

Then, we conclude that for larger r, the electric potential due to a quadrupole is inversely proportional to the cube of the distance r, while due to an electric dipole, it is inversely proportional to the square of r and inversely proportional to the distance r for a monopole.

33. (i) Refer to text on page 68.

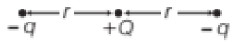
(ii) Equipotential surfaces when the electric field is in Z-direction.



The equipotential surfaces due to a single point charge is represented by concentric spherical shells of increasing radius, so they are not equidistant.

(iii) No, the electric field does not exist tangentially to an equipotential surface because no work is done in moving a charge from one point to other on equipotential surface. This indicates that the component of electric field along the equipotential surface is zero. Hence, the equipotential surface is perpendicular to field lines.

34. Let the three charges be located as shown in the figure.



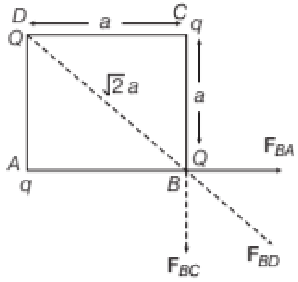
The potential energy of the system be

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)Q}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(-q)}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)(-q)}{2r}$$

As, $\frac{1}{4\pi\varepsilon_0}\left(\frac{-qQ}{r} - \frac{qQ}{r} + \frac{q^2}{2r}\right) = 0$

$$\Rightarrow \quad \frac{2qQ}{r} = \frac{q^2}{2r} \Rightarrow \frac{Q}{q} = \frac{1}{4} = 1:4$$

35. (a) Force acting on charge Q placed at point B, is due to charges placed at points A, C and D.



Here, magnitude of force on charge at point B due to charge at point A is

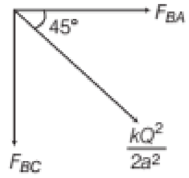
$$F_{BA} = \frac{kQq}{a^2}$$

Similarly, magnitude of force on charge at point B due to charge at point C is

$$F_{BC} = \frac{kQq}{a^2}$$

Also, the magnitude of force on charge at point B due charge at point D is

$$F_{BD} = \frac{kQ^2}{(\sqrt{2}a)^2} = \frac{kQ^2}{2a^2}$$



Let F is resultant of F_{BA} and F_{BC}.

$$\therefore \quad F = \sqrt{2}\cdot\frac{kQq}{a^2} \quad \left[\text{as } F_{BA} = F_{BC} = \frac{kQq}{a^2}\right]$$

$\therefore$ The resultant electric force on charge Q is

$$F_{net} = F + \frac{kQ^2}{2a^2} = \sqrt{2}\,\frac{kQq}{a^2} + \frac{kQ^2}{2a^2}$$

$$= \frac{kQ}{a^2}\left(\sqrt{2}q + \frac{Q}{2}\right)\text{newton}$$

(b) The potential energy of the system is given by

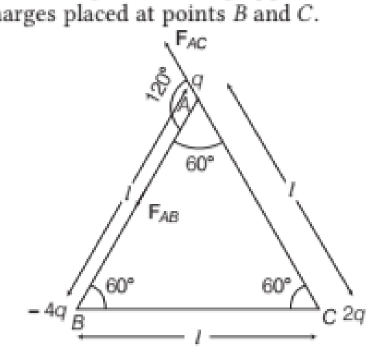
$$U = U_{AB} + U_{BC} + U_{CD} + U_{DA} + U_{AC} + U_{BD}$$

$$= \frac{kQq}{a} + \frac{kQq}{a} + \frac{kQq}{a} + \frac{kQq}{a} + \frac{kq^2}{\sqrt{2}a} + \frac{kQ^2}{\sqrt{2}a}$$

$$= \left[4\left(\frac{kQq}{a}\right) + \frac{kq^2}{\sqrt{2}a} + \frac{kQ^2}{\sqrt{2}a}\right]$$

Or

(a) Force acting on the charge q placed at A, is due to the charges placed at points B and C.



From the given figure, magnitude of force on charge at A due to charge at point C is given as

$$F_{AC} = \frac{k(q)(2q)}{l^2}, \text{ say} = F$$

Similarly, magnitude of force on charge at point A, due to charge at point B is

$$F_{AB} = \frac{k(4q)q}{l^2}, \text{ say} = 2F \quad (\because F_{AB} = 2F_{AC})$$

$$\therefore \quad F_{res} = \sqrt{F^2 + (2F)^2 + 2(F)(2F)\cos 120^\circ}$$

$$= \sqrt{F^2 + 4F^2 + 4F^2\left(-\frac{1}{2}\right)}$$

$$\left(\because \cos 120^\circ = -\frac{1}{2}\right)$$

$$= \sqrt{F^2 + 2F^2}$$

$$= \sqrt{3}F$$

$$\therefore \quad F_{res} = \sqrt{3}\times\frac{2kq^2}{l^2}\text{ N}$$

(b) The amount of the work done to separate the charges at infinite = Potential energy of the system

$$\therefore \quad U = U_{AB} + U_{BC} + U_{AC}$$

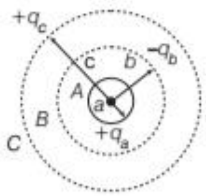
$$= \frac{k(-4q)q}{l} + \frac{k(-4q)(2q)}{l} + \frac{k(q)(2q)}{l}$$

$$= \frac{-4kq^2}{l} - \frac{8kq^2}{l} + \frac{2kq^2}{l}$$

$$U = \frac{-10kq^2}{l} \text{ J}$$

36. (i) Potential of three shells

At shell A



Potential, $V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{a} - \frac{q_b}{b} + \frac{q_c}{c}\right)$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right)\left[\because \sigma = \frac{q}{4\pi r^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

At shell B

Potential, $V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{b} - \frac{q_b}{b} + \frac{q_c}{c}\right)$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right)\left[\because \sigma = \frac{q}{4\pi r^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

At shell C

Potential, $V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{c} - \frac{q_b}{c} + \frac{q_c}{c}\right)$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^2\sigma}{c} - \frac{4\pi b^2\sigma}{c} + \frac{4\pi c^2\sigma}{c}\right)\left[\because \sigma = \frac{q}{4\pi r^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2 + c^2}{c}\right)$$

(ii) Relation between the radii

Now, $V_A = V_C$ (given)

$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\frac{(a^2 - b^2 + c^2)}{c}$$

$$a - b + c = \frac{a^2 - b^2 + c^2}{c} = \frac{a^2 - b^2}{c} + c$$

$$c(a - b) = a^2 - b^2$$

$$\Rightarrow \quad c = a + b \; [\because (a^2 - b^2) = (a - b)(a + b)]$$

37. Radius of sphere $A = R$

Surface charge density on sphere $A = \sigma$

Radius of sphere $B = 2R$

Surface charge density on sphere $B = \sigma$

Before contact, the charge on sphere A is

$Q_1 =$ Surface charge density $\times$ Surface area

$\Rightarrow \quad Q_1 = \sigma \cdot 4\pi R^2$ ...(i)

Before contact, the charge on sphere B is

$Q_2 =$ Surface charge density $\times$ Surface area

$Q_2 = \sigma \cdot 4\pi(2R)^2 = \sigma \cdot 16\pi R^2$ ...(ii)

Let after the contact, the charge on A be Q_1' and the charge on B be Q_2'.

According to the conservation of charge, the charge before contact is equal to charge after contact.

$$Q_1' + Q_2' = Q_1 + Q_2$$

Now, from Eqs. (i) and (ii), we get

$$Q_1' + Q_2' = 4\pi R^2\sigma + 16\pi R^2\sigma$$

$$= 20\pi R^2\sigma \quad \text{...(iii)}$$

As they are in contact. So, they have same potential.

Potential on sphere A is $V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1'}{R}$

Potential on sphere B is $V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2'}{2R}$

So, $V_A = V_B$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2'}{2R}$$

$$\Rightarrow \quad \frac{Q_1'}{R} = \frac{Q_2'}{2R}$$

$$\Rightarrow \quad 2Q_1' = Q_2' \quad \text{...(iv)}$$

Putting the value of Q_2' in Eq. (iii), we get

$$Q_1' + 2Q_1' = 20\pi R^2\sigma \Rightarrow 3Q_1' = 20\pi R^2\sigma$$

$$\Rightarrow \quad Q_1' = \frac{20}{3}\pi R^2\sigma$$

and $Q_2' = \frac{40}{3}\pi R^2\sigma$ [from Eq. (iv)]

Let the new charge densities be σ_1 and σ_2.

$$\sigma_1 = \frac{Q_1'}{4\pi R^2} = \frac{20\pi R^2\sigma}{3 \times 4\pi R^2} = \frac{5}{3}\sigma$$

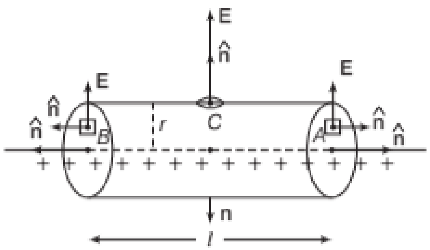
$$\sigma_2 = \frac{Q_2'}{4\pi(2R)^2} = \frac{40\pi R^2\sigma}{3 \times 4\pi \times 4R^2} = \frac{40\sigma}{16 \times 3}$$

$$\sigma_2 = \frac{10\sigma}{4 \times 3} = \frac{5}{6}\sigma$$

Thus, the surface charge densities on spheres after contact are $\frac{5}{3}\sigma$ and $\frac{5}{6}\sigma$.

38. (a) **Field due to an infinitely long thin straight charged line**

Consider an infinitely long thin straight line with uniform linear charge density (λ).



Gaussian surface for a long thin straight line of uniform charge density

From symmetry, the electric field is everywhere radial in the plane cutting the wire normally and its magnitude only depends on the radial distance (r).

From Gauss' law,

$$\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

Now, $\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \oint_S \mathbf{E}\cdot\hat{\mathbf{n}}\, dS$

$$= \oint_A \mathbf{E}\cdot\hat{\mathbf{n}}\, dS + \oint_B \mathbf{E}\cdot\hat{\mathbf{n}}\, dS + \oint_C \mathbf{E}\cdot\hat{\mathbf{n}}\, dS$$

$$\therefore \oint_S \mathbf{E}\cdot dS = \oint_A \mathbf{E}\, dS\cos 90^\circ + \oint_B \mathbf{E}\, dS\cos 90^\circ + \oint_C \mathbf{E}\, dS\cos 0^\circ$$

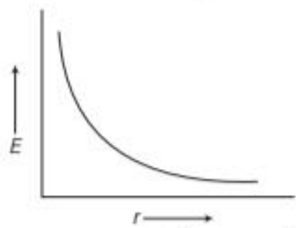
$$= \oint_C E\, dS = E(2\pi r l)$$

Charge enclosed in the cylinder, $q = \lambda l$

$$\therefore E(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of the electric field is radially outward from the positive line charge. For negative line charge, it will be radially inward.

(b) Electric field (E) due to the linear charge is inversely proportional to the distance (r) from the linear charge. The variation of electric field (E) with distance (r) is shown in figure.



(c) $$v = \int \mathbf{E}\cdot d\mathbf{r} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r}\, dr = \frac{\lambda}{2\pi\varepsilon_0}\cdot\int_{r_1}^{r_2}\frac{1}{r}\, dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\log\frac{r_2}{r_1}\right]$$

$$\text{Work done } = qv = q\left[\frac{\lambda}{2\pi\varepsilon_0}\left(\log\frac{r_2}{r_1}\right)\right]$$

39. Work done, $W = q \times \Delta V$

But $\Delta V = 0$ as the two diagonally opposite points are at the same potential due to 10μC charge.

$\therefore \quad W = 2\mu\text{C} \times 0 = 0$

Work done $\quad W = 0$

40. Given, $\quad r = 0.1$ m, $V = +50$ V and $q = ?$

As, $\quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$

$\Rightarrow \quad 50 = 9\times 10^9 \times \frac{q}{0.1}$

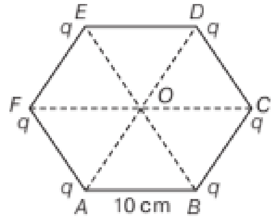
$\therefore \quad q = \frac{50 \times 0.1}{9 \times 10^9} = 5.6\times 10^{-10}$ C

As, V is positive, therefore, q must be positive.

41. Refer to Example 6 on page 65.

Ans. At 6 cm from charge -3×10^{-8} C.

42. $ABCDEF$ is a regular hexagon of side 10 cm each. At each corner, the charge $q = 5\,\mu$C is placed. O is the centre of the hexagon.



Given, $AB = BC = CD = DE = EF = FA = 10$ cm

As, the hexagon has six equilateral triangles, so the distance of centre O from every vertex is 10 cm.

i.e. $OA = OB = OC = OD = OE = OF = 10$ cm

$\therefore$ Potential at point O = Sum of potentials at centre O due to individual point charge

i.e. $V_O = V_A + V_B + V_C + V_D + V_E + V_F$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q}{OA}+\frac{q}{OB}+\frac{q}{OC}+\frac{q}{OD}+\frac{q}{OE}+\frac{q}{OF}\right]$$

$$\left[\because V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}\right]$$

Putting the values, we get

$$V_O = 9\times 10^9\left[\frac{5\times 10^{-6}}{10\times 10^{-2}} + \frac{5\times 10^{-6}}{10\times 10^{-2}} + \frac{5\times 10^{-6}}{10\times 10^{-2}} + \frac{5\times 10^{-6}}{10\times 10^{-2}} + \frac{5\times 10^{-6}}{10\times 10^{-2}} + \frac{5\times 10^{-6}}{10\times 10^{-2}}\right]$$

$$= 9 \times 10^9 \times \frac{6 \times 10^{-6} \times 5}{10 \times 10^{-2}}$$

$$= 27 \times 10^5$$

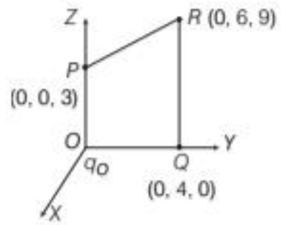
$$= 27 \times 10^6 \text{ V}$$

43. Charge q_O at origin $O = 8 \text{ mC} = 8 \times 10^{-3} \text{C}$

Charge q_P at point $P = -2 \times 10^{-9}$ C

Distance $OP = r_P = 3 \text{ cm} = 0.03$ m

Distance, $OQ = r_Q = 4$ cm $= 0.04$ m



Work done in bringing the charge q_P from P to Q

$= q \times$ potential difference between Q and P

$$W_{PQ} = q(V_Q - V_P)$$

$$= -2 \times 10^{-9}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_O}{OQ} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_O}{OP}\right)$$

$$= -2 \times 10^{-9}\left(\frac{9 \times 10^9 \times 8 \times 10^{-3}}{0.04} - \frac{9 \times 10^9 \times 8 \times 10^{-3}}{0.03}\right)$$

$$= \frac{18 \times 8 \times 10^{-3} \times 0.01}{0.0012}$$

$$= 1.2 \text{ J}$$

Thus, the work done in bringing the charge of -2×10^{-9}C from P to Q is 1.2 J.

44. Potential at the centre,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$$

$$= 9 \times 10^9 \times 10^{-8} \times \frac{60}{360} \times 2\pi r$$

$$= 9 \times 10^{+9} \times 10^{-8} \times \frac{2 \times 3.14 \times 2}{6} = 188.4 \text{ V}$$

45. Refer to Example 11 on pages 69 and 70.

[**Ans.** 6.5×10^{-16}kg]

46. Surface charge density, $\sigma = 10^{-8}$ C/m^2

Potential difference of two equipotential surface,

$$dV = 5 \text{ V}$$

Let the separation between two equipotential surfaces be dr. Electric field intensity E due to infinite plane sheet is given by

$$\text{E} = \frac{\sigma}{\varepsilon_0}$$

The relation between E and V is given by

$$\text{E} = \frac{dV}{dr} \Rightarrow \frac{dV}{dr} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow \quad dr = \frac{2\varepsilon_0 \cdot dV}{\sigma} = \frac{2 \times (8.85 \times 10^{-12}) \times 5}{10^{-8}}$$

$$= 8.85 \times 10^{-3} \text{ m}$$

47. (i) $\because$ Electric field intensity and potential difference are related as,

$$E = -\frac{\Delta V}{\Delta r}$$

$$\Rightarrow \quad \Delta V = -E\,\Delta r$$

By Pythagoras law, $AC^2 = AB^2 + BC^2$

$$\Rightarrow \quad 5^2 - 3^2 = AB^2$$

$$\Rightarrow \quad AB = \Delta r = 4$$

$$\Rightarrow \quad V_A - V_C = -4E$$

$$\Rightarrow \quad V_C - V_A = 4E$$

(ii) As, $V_C - V_A = 4E$, is positive.

$\therefore \quad V_C > V_A$

Potential is greater at point C than at point A, as potential decreases along the direction of electric field.

48. (i) Electric field between the plates is given by

$$E = \frac{\Delta V}{\Delta x} = -\frac{(V_B - V_A)}{1 \times 10^{-2}}$$

$$= \frac{-(0-10)}{10^{-2}} = 10^3 \text{ V/m}$$

It is directed from A to B.

(ii) Work done in moving a charge from Y to Z is

$$W_{Y\text{-}Z} = q(\Delta V) = 20 \times 10^{-6}(V_Z - V_Y)$$

$$= 20 \times 10^{-6}(0 - 10)$$

$$= -20 \times 10^{-5} \text{ J}$$

|TOPIC 2|
Dielectric and Capacitance

In this topic, we are going to learn about characteristic properties of conductors and insulators. Also we will go through the concepts of capacitors and their combinations.

CONDUCTORS AND INSULATORS

Let us discuss some characteristics of conductors and insulators as discussed below.

Conductors

Conductors are the materials through which electric charge can flow easily. Most of the metals are conductors of electric charge. Silver is the best conductor of electric charge.

Under electrostatic conditions, the conductors have following properties

(i) Inside a conductor, electrostatic field is zero.

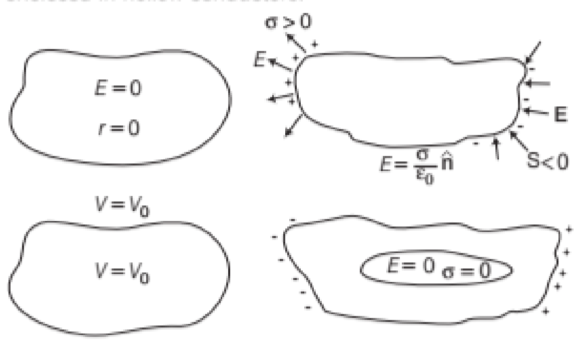
(ii) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.

(iii) The interior of the conductor can have no excess charge in the static situation.

(iv) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.

(v) Surface charge density of a conductor could be different at different points.

> ***Electrostatic Shielding***
>
> The phenomenon of protecting a certain region of space from external electric field is called electrostatic shielding. We know that inside a conductor, electric field is zero, so to protect some instruments from external field, they are enclosed in hollow conductors.



Insulators

Insulators are the materials through which electric charge cannot flow e.g. glass, rubber, wood, etc. Insulators are also called **dielectrics**, when an electric field is applied, induced charges appear on the surface of the dielectric. Hence, it can be said that dielectrics are the insulating materials which transmit the electric effect without conducting.

Free Charges and Bound Charges Inside the Conductor

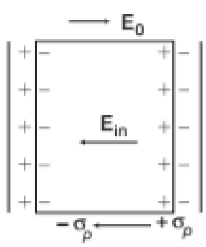
In metallic conductors, electrons are the charge carriers. In a metal, the outer (valence) electrons part away from their atoms and are free to move, these electrons are called **free electrons** or **conduction electrons**. The electrical conductivity of a material depends upon the number of free electrons present in it. Materials which have high number of free electrons are good conductors and which have less number of free electrons are bad conductors.

When an electron leaves an atom, atom becomes positively charged ion. The positively charged ions and bound electrons remain held in their fixed positions and are called **bound charges**.

Dielectrics and Polarisation

Dielectrics (or insulators) are non-conducting substances. In contrast to conductors, they have no (or negligible number of) free charges or charge carriers.

In a dielectric under the effect of an external field, a net dipole moment is induced in the dielectric. Due to molecular dipole moments, a net charge appears on the surface of the dielectric.



Polarised dielectric slab

These induced charges (of densities $-\sigma_p$ and and $+\sigma_p$) produce a field opposing the external field. Induced field is lesser in magnitude than the external field. So, field inside the dielectric gets reduced.

$$E = |E_0| - |E_{in}|$$

where, E = resultant electric field in the dielectric,

E_0 = external electric field between two plates

and E_{in} = electric field inside the dielectric.

A net dipole moment is developed by an external field in either case, whether a polar or non-polar dielectric.

Dielectric Constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of the electric field on placing the dielectric between the two plates is called the dielectric constant of the dielectric medium.

It is also known as **relative permittivity** or **specific inductive capacity** and is denoted by K (or ε_r).

Therefore, dielectric constant of a dielectric medium is given by

$$K = \frac{E_0}{E}$$

Note The value of K is always greater than 1.

Polarisation (P)

The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called polarisation. It is denoted by P. If p is induced dipole moment acquired by an atom of the dielectric and N is the number of atoms per unit volume, then polarisation is given by $\boxed{P = Np}$

The induced dipole moment (p) acquired by the atom is found to be directly proportional to the reduced value of electric field (E) and is given by

$$p = \alpha\varepsilon_0 \mathbf{E}$$

where, α is constant of proportionality and is called **atomic polarisability.**

Electric Susceptibility (χ)

The polarisation density of a dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as $\boxed{P = \chi\varepsilon_0 E}$

The constant of proportionality χ is called **electric susceptibility** of the dielectric slab. It is a dimensionless constant.It describes the electrical behaviour of a dielectric. It has different values for different dielectrics.

For vacuum, $\chi = 0$

Relation between dielectric constant and electric susceptibility can be given as

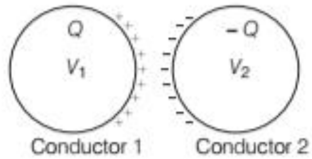
$$K = 1 + \chi$$

Dielectric Strength

The maximum electric field that a dielectric can withstand without breakdown (of its insulating property), is called its dielectric strength. For air, it is about 3×10^6 V/m.

Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulating medium. The conductors have charges Q and $-Q$ with potential difference, $V = V_1 - V_2$ between them. The electric field in the region between the conductors is proportional to the charge Q.



A system of two conductors or capacitors

If the potential difference (V) is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field, then V is proportional to Q and the ratio $\frac{Q}{V}$ is a constant.

$$C = \frac{Q}{V}$$

The constant C is called the capacitance of the capacitor. Capacitance C depends on shape, size and separation of the system of two conductors.The SI unit of capacitance is farad. Its dimensional formula is $[M^{-1}L^{-2}T^4A^2]$.

$$1 \text{ farad} = 1 \text{ coulomb/volt}$$

A capacitor with fixed capacitance is symbolically shown as ⊣⊢, while the one with variable capacitance is shown as ⊣̸⊢. In practice, farad is a very big unit, the most common units are its sub-multiples.

$$1\,\mu F = 10^{-6}\text{ F}, 1\text{ nF} = 10^{-9}\text{ F}, 1\text{ pF} = 10^{-12}\text{ F}$$

EXAMPLE |1| When 1×10^{12} electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Find the capacitance of the two conductors.

Sol. Given, number of electrons,

$$n = 1 \times 10^{12}$$

$\therefore$ Charge transferred,

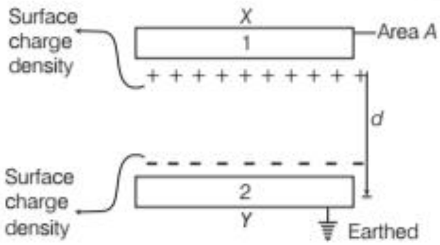
$$Q = ne = 1 \times 10^{12} \times 1.6 \times 10^{-19}$$
$$= 1.6 \times 10^{-7}\text{ C} \qquad [\because e = 1.6 \times 10^{-19}\text{ C}]$$

$\therefore$ Capacitance between two conductors,

$$C = \frac{Q}{V} = \frac{1.6 \times 10^{-7}}{10}$$
$$= 1.6 \times 10^{-8}\text{ F}$$

PARALLEL PLATE CAPACITOR

Parallel plate capacitor consists of two thin conducting plates each of area A held parallel to each other at a suitable distance d. One of the plates is insulated and other is earthed. And also there is vacuum between the plates.



Suppose the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges.

Suppose the surface density of charge on each plate is σ. We know that the intensity of electric field at a point between two plane, parallel sheets of equal and opposite charges is σ/ε_0, where ε_0 is the permittivity of free space.
The intensity of electric field between the plates will be given by

$$E = \frac{\sigma}{\varepsilon_0}$$

The charge on each plate is q and the area of each plate is A. Thus,

$$\sigma = \frac{q}{A} \text{ and so, } E = \frac{q}{\varepsilon_0 A} \qquad \text{...(i)}$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed$$

Substituting the value of E from Eq. (i), we get

$$V = \frac{qd}{\varepsilon_0 A}$$

$\therefore$ Capacitance of the parallel plate capacitor is given by

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} \text{ or } \boxed{C = \frac{\varepsilon_0 A}{d}}$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

It is clear from this formula that in order to obtain high capacitance.

(i) A should be large, i.e. the plates of large area should be taken.

(ii) d should be small, i.e. the plates should be kept close to each other.

Note Capacity of an isolated spherical conductor is

$$\boxed{C = 4\pi\varepsilon_0 r}$$

where, r = radius of the sphere.

Leakage of Charge from a Capacitor

From the formula $C = q/V$, it is clear that for large C, V is small for a given q. This means a capacitor with large capacitance can hold large amount of charge q at small V. This is very important fact, because the large amount of charge implies strong electric field around the conductor.

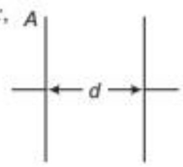
This strong electric field can ionise the surrounding air and accelerate the charges, so produced to oppositely charged plates, thereby neutralising the charge on the capacitor plates. This means the charge of the capacitor leaks away due to the reduction in insulating power of the intervening medium.

EXAMPLE |2| What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? (You will realise from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors). **NCERT**

Sol. Given, capacitance, $C = 2$ F

and separation between plates, $d = 0.5 \text{ cm} = 0.5 \times 10^{-2}$ m

Capacitance of a parallel plate capacitor,



$$C = \frac{\varepsilon_0 A}{d}$$

$$\text{or } A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

$$= 1.13 \times 10^9 \text{ m}^2$$

$$= 1130 \text{ km}^2$$

This area is very large, so it is not possible that the capacitance of a capacitor is too large as 2 F.
So, the capacitance of any capacitor should be the range of 2 µF.

EXAMPLE |3| A parallel plate capacitor has plate area 25 cm^2 and a separation of 2 mm between the plates. The capacitor is connected to a battery of 12 V.

(i) Find the charge on the capacitor.

(ii) If the plate separation is decreased to 1.0 mm, then find the extra charge given by the battery to the positive plate.

Sol. Given, area of plate, $A = 25\text{ cm}^2 = 25\times10^{-4}\text{ m}^2$

Distance between the plates, $d = 2\text{ mm} = 2\times10^{-3}\text{ m}$

Potential difference, $V = 12\text{ V}$

(i) Charge on the capacitor, $q = CV$

$$= \frac{\varepsilon_0 A}{d}V = \frac{8.85\times10^{-12}\times25\times10^{-4}\times12}{2\times10^{-3}}$$

$$= 1.33\times10^{-10}\text{ C}$$

(ii) If the plate separation is decreased to half, the capacity becomes twice. Then, charge becomes twice as battery is still connected.

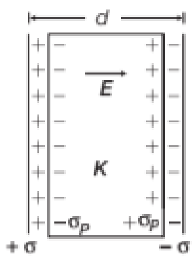
$\therefore$ Extra charge given by the battery $= q' - q$

$$= 2q - q = q = 1.33\times10^{-10}\text{ C}$$

Effect of Dielectric on Parallel Plate Capacitor

Consider a dielectric is inserted between the plates of a parallel plate capacitor and fully occupying the intervening region as shown in figure. The dielectric is polarised by the field, with surface charge densities σ_p. and $-\sigma_p$.

The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$.



Dielectric between the plates of a capacitor

So, net electric field between the plates, $E = \dfrac{\sigma - \sigma_p}{\varepsilon_0}$

[$\because$ dielectric is polarised in the opposite direction of external field]

$\therefore$ Potential difference between the plates,

$$V = Ed = \frac{\sigma - \sigma_p}{\varepsilon_0}d$$

For linear dielectrics, we expect σ_p to be proportional to E_0 i.e. to σ.

Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write,

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

where, K is a constant characteristics of the dielectric.

Clearly, $K > 1$ [$\because \sigma_p < \sigma$]

then, $V = \dfrac{\sigma d}{\varepsilon_0 K} = \dfrac{qd}{A\varepsilon_0 K}$

$\therefore$ The capacitance C with dielectric between the plates is given by

$$C = \frac{q}{V} = \frac{\varepsilon_0 KA}{d}$$

The product $\varepsilon_0 K$ is called the **permittivity of the medium** and is denoted by ε.

$$\varepsilon = \varepsilon_0 K$$

For vacuum, $K = 1$ and $\varepsilon = \varepsilon_0$, where ε_0 is called the **permittivity of the vacuum**.

The dimensionless ratio,

$$K = \frac{\varepsilon}{\varepsilon_0}$$

is called the **dielectric constant** of the substance.

Similarly, $K = \dfrac{C}{C_0}$

Thus, the dielectric constant of a substance is the factor ($K > 1$) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

(i) When a dielectric slab of thickness t is inserted between the plates, then

Capacitance, $C = \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}$

(ii) If several slabs of dielectric constants $K_1, K_2, K_3, \ldots$ and respective thicknesses $t_1, t_2, t_3, \ldots$ are placed in between the plates of a capacitor, then capacitance,

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + \ldots) + \dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3} + \ldots}$$

(iii) If a metallic slab ($K = \infty$) of thickness t is placed between the plates of capacitor, then

Capacitance,

$$C = \frac{\varepsilon_0 A}{d - t}.$$

If metallic slab fills the entire space between the plates (i.e. $d = t$), then capacitance will become infinite.

EXAMPLE |4| In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} m^2$ and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

(ii) If this capacitor is connected to 100 V supply, what would be the charge on each plate?

(iii) How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected?

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Sol. Given, area of each plate, $A = 6 \times 10^{-3} m^2$

Distance between the plates,

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

(i) Capacitance of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$\therefore \quad C = 1.77 \times 10^{-11} F$

(ii) Charge on parallel plate capacitor is given by

$$Q = CV = 1.77 \times 10^{-11} \times 100 = 1.77 \times 10^{-9} C$$

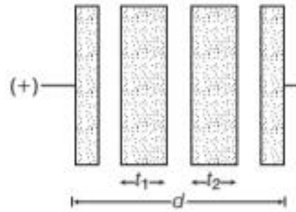
(iii) Given, $K = 6$

Now, $C' = KC$

$\Rightarrow \quad \frac{Q'}{V} = \frac{KQ}{V}$

$\therefore \quad Q' = KQ = 6 \times 1.77 \times 10^{-9} = 10.62 \times 10^{-9} C$

EXAMPLE |5| An air-cored capacitor of plate area A and separation d has a capacity C. Two dielectric slabs are inserted between its plates in two different manners as shown. Calculate the capacitance in it.



Sol. Let the charges on the plates are Q and $-Q$.

Electric field in free space is $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$.

Electric field in first slab is $E_1 = \frac{E_0}{K_1} = \frac{Q}{A\varepsilon_0 K_1}$.

Electric field in second slab is $E_2 = \frac{E_0}{K_2} = \frac{Q}{A\varepsilon_0 K_2}$.

The potential difference between the plates is

$$V = E_0(d - t_1 - t_2) + E_1 t_1 + E_2 t_2$$

$$\Rightarrow V = E_0\left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}\right)$$

$$\left[\because E_1 = \frac{E_0}{K_1} \text{ and } E_2 = \frac{E_0}{K_2}\right]$$

$$\therefore V = \frac{Q}{A\varepsilon_0}\left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}\right)$$

$$\therefore C = \frac{\varepsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}}$$

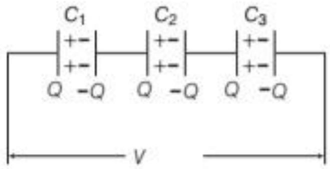
COMBINATION OF CAPACITORS

When there is a combination of capacitors in a circuit, we can sometimes replace that combination with an equivalent capacitor, i.e. single capacitor, that has the same capacitance as the actual combination of capacitors has with such a replacement, that we can simply find the circuit, affording easier solutions for unknown quantities of the circuit.

Here, we discuss two basic combinations of capacitors which can be replaced by single equivalent capacitor.

Capacitors in Series

When a potential difference (V) is applied across several capacitors connected end to end in such a way that, sum of potential differences across all the capacitors is equal to the applied potential difference V, then these capacitors are said to be connected in series.



Series combination of capacitors

The potential difference across the separate capacitors are given by

$$V_1 = \frac{Q}{C_1}, \; V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

However, the potential difference across the series combination of capacitors is V volt

where, $\quad V = V_1 + V_2 + V_3 \quad$ ...(i)

Let C_S represents the equivalent capacitance, then

$$V = \frac{Q}{C_S} \quad \text{...(ii)}$$

Combining Eqs. (i) and (ii), we get

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The equivalent capacitance of n capacitors connected in series is equal to the sum of the reciprocals of individual capacitances of the capacitors.

Mathematically, it is expressed as,

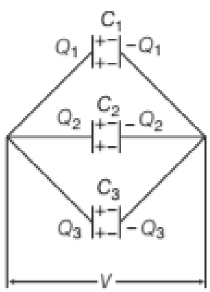
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

All the capacitors connected in series have same amount of charge, but potential differences between their plates are inversely proportional to their capacitances. This combination is used when a high voltage is to be divided on several capacitors. Here, capacitor with minimum capacitance has maximum potential difference between the plates.

Capacitors in Parallel

Capacitors are said to be connected in parallel when a potential difference that is applied across their combination results in the potertial differnce same across each capacitor.

When a potential difference (V) is applied across several capacitors connected in parallel, then the potential difference (V) exists across each capacitor. The total charge (Q) stored on the capacitor is the sum of the charges stored on all the capacitors.



Parallel combination of capacitors

If Q is the total charge on the parallel network, then

$$Q = Q_1 + Q_2 + Q_3 \quad \ldots(i)$$

Let C_P be the equivalent capacitance of the parallel combination, then

$$Q = C_P V,\ Q_1 = C_1 V,\ Q_2 = C_2 V$$

and $$Q_3 = C_3 V \quad \ldots(ii)$$

Combining Eqs. (i) and (ii), we obtain

$$C_P V = C_1 V + C_2 V + C_3 V$$

$$\Rightarrow \quad C_P = C_1 + C_2 + C_3$$

The equivalent capacitance of n number of capacitors in parallel is equal to the algebraic sum of the individual capacitances of the capacitors.

Mathematically, it is expressed as,

$$C_{eq} = C_1 + C_2 + C_3 + \ldots + C_n$$

All the capacitors connected in parallel have same potential difference between their plates but the charge is distributed proportionally to their capacitances.

Capacitors are combined in parallel, when we require a large capacitance at small potential.

EXAMPLE |6| Three capacitors each of capacitance 9 pF are connected in series.

(i) What is the total capacitance of the combination?

(ii) What is the potential difference across each capacitor, if the combination is connected to a 120 V supply? **NCERT**

Sol. There are three capacitors each of capacitance 9 pF.

$$\therefore \quad C_1 = C_2 = C_3 = 9\ \text{pF}$$

and voltage, $V = 120$ V

(i) The total capacitance in series combination,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$

$$\Rightarrow \quad \frac{1}{C_S} = \frac{3}{9} \quad \Rightarrow \quad C_S = 3\ \text{pF}$$

(ii) Let the charge across the system be q and potentials across C_1, C_2 and C_3 be V_1, V_2 and V_3, respectively.

Charge, $q = C_S \cdot V = 3 \times 120 = 360$ pC

Potential difference across C_1,

$$V_1 = \frac{q}{C_1} = \frac{360}{9} = 40\ \text{V}$$

Potential difference across C_2,

$$V_2 = \frac{q}{C_2} = \frac{360}{9} = 40\ \text{V}$$

Potential difference across C_3,

$$V_3 = \frac{q}{C_3} = \frac{360}{9} = 40\ \text{V}$$

Thus, the potential difference across each capacitor is 40 V.

EXAMPLE |7| It is required to construct a 10 µF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 µF are available but they withstand only 50 V. Design a combination which can yield the desired result.

Sol. Capacitor of 10µF can withstand only 50 V, therefore to be connected across a 200 V battery, four capacitors must be connected in series in a row. Capacitor C_1 of each row of four capacitors is

$$\frac{1}{C_1} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$$

$$\Rightarrow \quad C_1 = \frac{10}{4} = 2.5\ \mu\text{F}$$

For a total capacity of 10 µF, four such rows of capacitors must be connected in parallel, so that

$$C_P = 4\ C_1$$

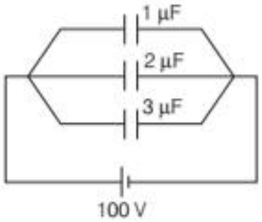
$$= 4 \times 2.5 = 10\ \mu\text{F}$$

Hence, we need 16 capacitors with 4 capacitors in series in each row and 4 such rows in parallel.

EXAMPLE |8| In the circuit shown in figure, find

(i) the equivalent capacitance and

(ii) the charge stored in each capacitor.



Sol. (i) The capacitors are in parallel. Hence, the equivalent capacitance is

$$C = C_1 + C_2 + C_3$$
$$= (1 + 2 + 3) = 6\,\mu F$$

(ii) Total charge drawn from the battery,

$$q = CV = 6 \times 100\,\mu C$$
$$= 600\mu C$$

This charge will be distributed in the ratio of their capacities. Hence,

$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3$

$$\therefore \quad q_1 = \left(\frac{1}{1+2+3}\right) \times 600 = 100\,\mu C$$

$$q_2 = \left(\frac{2}{1+2+3}\right) \times 600 = 200\,\mu C$$

and $$q_3 = \left(\frac{3}{1+2+3}\right) \times 600 = 300\,\mu C$$

EXAMPLE |9| Three capacitors of $1\mu F$, $2\mu F$ and $3\mu F$ are joined in series.

(i) How many times will the capacity become when they are joined in parallel ?

(ii) Determine the charge supplied by the battery of 100 V to the maximum resultant capacitor among both the arrangements.

Sol. (i) Given, $C_1 = 1\,\mu F$, $C_2 = 2\mu F$, $C_3 = 3\,\mu F$

The combined capacity (C_S) in series combination is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$\Rightarrow \quad C_S = \frac{6}{11}\,\mu F$$

The combined capacity (C_P) in parallel combination is given by

$$C_P = C_1 + C_2 + C_3 = 1 + 2 + 3 = 6\,\mu F$$

$$\Rightarrow \quad C_P = 11 C_S$$

(ii) As, $\quad C_P > C_S$

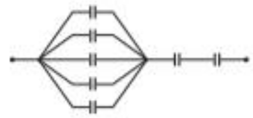
$\therefore$ The charge supplied by 100 V battery,

$$q_p = C_P V = 6\mu F \times 100 = 6 \times 10^{-6} \times 100$$

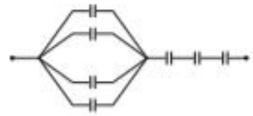
$$\Rightarrow \quad q_p = 6 \times 10^{-4}\,C = 600\,\mu C$$

EXAMPLE |10| Seven capacitors each of capacitance $2\,\mu F$ are connected in a configuration to obtain an effective capacitance $\frac{10}{11}\mu F$. Which of the following combinations will achieve the desired result?

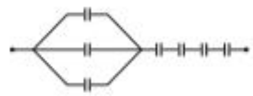
(i)



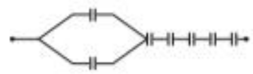
(ii)



(iii)



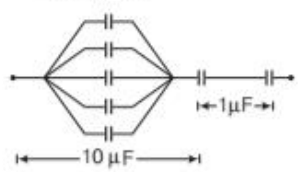
(iv)



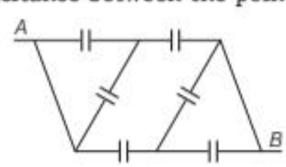
Sol. Consider the first configuration, we have

In series, $C = \dfrac{C_1 C_2}{C_1 + C_2}$

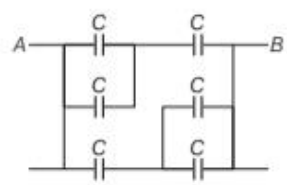
$$\therefore \quad C_{net} = \frac{(10)(1)}{10+1} = \frac{10}{11}\,\mu F$$



EXAMPLE |11| A network of six identical capacitors, each of value C is made as shown in the figure. Find the equivalent capacitance between the points A and B.



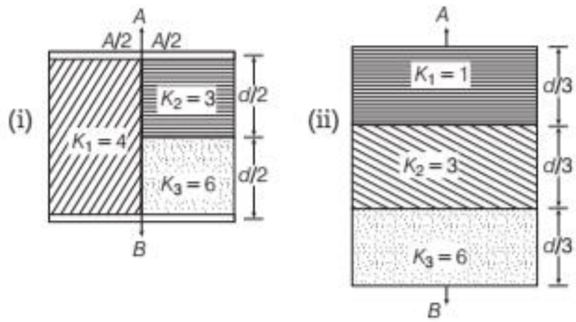
Sol. The equivalent network of the given network is shown below.



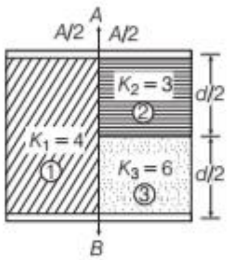
Therefore equivalent capacitance,

$$C_{eq} = [2\,C \text{ series } C] \| [C \text{ series } 2C]$$
$$= 2\left[\frac{2C \times C}{2C + C}\right] = \frac{4C}{3}$$

EXAMPLE |12| Find the equivalent capacitance between A and B. Given area of each plate $= A$ and separation between plate $= d$.



Sol. (i)



Capacitance, $C_1 = \dfrac{K_1 \dfrac{A}{2} \varepsilon_0}{d} = \dfrac{4 \dfrac{A}{2} \varepsilon_0}{d} = \dfrac{2A\varepsilon_0}{d}$

$\left[\because C = \dfrac{KA\varepsilon_0}{d}\right]$

Capacitance, $C_2 = \dfrac{K_2 \dfrac{A}{2} \varepsilon_0}{\dfrac{d}{2}} = \dfrac{3A\varepsilon_0}{d}$

Capacitance, $C_3 = \dfrac{K_3 \dfrac{A}{2} \varepsilon_0}{\dfrac{d}{2}} = \dfrac{6A\varepsilon_0}{d}$

C_2 and C_3 are in series, $C' = \dfrac{C_2 C_3}{C_2 + C_3} = \dfrac{2A\varepsilon_0}{d}$

C' and C_1 are in parallel $= \dfrac{4A\varepsilon_0}{d}$

(ii) Capacitance, $C_1 = \dfrac{K_1 A\varepsilon_0}{d/3} = \dfrac{3A\varepsilon_0}{d}$,

$C_2 = \dfrac{K_2 A\varepsilon_0}{d/3} = \dfrac{9A\varepsilon_0}{d}$

and $C_3 = \dfrac{K_3 A\varepsilon_0}{d/3} = \dfrac{18A\varepsilon_0}{d}$

$\because C_1, C_2$ and C_3 are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$\therefore$ Equivalent capacitance, $C_{eq} = \dfrac{2A\varepsilon_0}{d}$

ENERGY STORED IN A CAPACITOR

The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential. Let us assume that initially both the plates are uncharged. Now, we have to repeatedly move small positive charges from one plate and transfer them to the other plate.

Now, when an additional small charge (dq) is transferred from one plate to another plate, the small work done is given by $\quad dW = V'dq = \dfrac{q'}{C} dq$

[$\because$charge on plate when dq charge is transferred be q']

The total work done in transferring charge Q is given by

$$W = \int_0^Q \frac{q'}{C} dq = \frac{1}{C} \int_0^Q q' dq$$

$$= \frac{1}{C} \left[\frac{(q')^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

This work is stored as electrostatic potential energy U in the capacitor.

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{(CV)^2}{2C} = \frac{1}{2} CV^2} \quad [\because Q = CV]$$

The energy stored per unit volume of space in a capacitor is called **energy density**.

Energy density, $\boxed{u = \dfrac{1}{2} \varepsilon_0 E^2}$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors.

i.e. $\quad U = U_1 + U_2 + U_3 + \ldots$

Change in Energy on Introducing a Dielectric Slab

(i) When a dielectric slab is inserted between the plates of a charged capacitor, with battery connected to its plates. Then, the capacitance becomes K (dielectric constant) times and energy stored in the capacitor becomes KU_0.

(ii) When a dielectric slab is inserted between the plates of a charged capacitor and battery is disconnected. Then, the charge on the plates remains unchanged and energy stored in the capacitor becomes $\dfrac{U_0}{K}$, i.e. energy decreases.

Note This topic has been frequently asked in previous years 2015, 2014, 2012, 2011, 2010.

EXAMPLE |13| A capacitor of capacity 10 μF is subjected to charge by a battery of 10 V. Calculate the energy stored in the capacitor.

Sol. Given, capacity, $C = 10\,\mu F = 10 \times 10^{-6}$ F

Voltage, $V = 10$ V, energy, $E = ?$

$\therefore$ Energy stored in the capacitor, $E = \frac{1}{2} CV^2$

$$= \frac{1}{2} \times 10 \times 10^{-6} \times 10 \times 10 = 5 \times 10^{-4} \text{J}$$

EXAMPLE |14| A parallel plate capacitor has plate area A and separation d. It is charged to a potential difference V_0. This charging battery is disconnected and the plates are pulled apart to three times the initial separation. Calculate the work required to separate the plates.

Sol. $\therefore$ Capacitance, $C = \frac{\varepsilon_0 A}{d}$

Charge on plate, $Q = CV = \frac{\varepsilon_0 A}{d} V_0$

Energy stored, $U = Q^2 / 2C$

As d is increased 3 times, so C decreases 3 times. Battery is disconnected, so Q remains same. The difference in the energy is the work done,

Change in potential energy

$$\Delta U = U_i - U_f$$

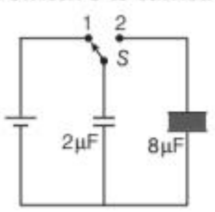
$$= \frac{Q^2}{2}\left[\frac{1}{C_i} - \frac{1}{C_f}\right] = \left(\frac{\varepsilon_0 A V_0}{d}\right)^2 \times \frac{1}{2}\left[\frac{1}{C} - \frac{1}{C/3}\right]$$

$$= -\frac{1}{2} \times \left(\frac{\varepsilon_0 A V_0}{d}\right)^2 \times \frac{2}{C} = \left(\frac{\varepsilon_0 A V_0}{d}\right) \times \frac{d}{\varepsilon_0 A V_0}$$

$$= -\frac{\varepsilon_0 A V_0}{d}$$

$\therefore$ Work done, $\Delta W = -\Delta U = \frac{\varepsilon_0 A V_0}{d}$

EXAMPLE |15| A 2μF capacitor is charged as shown in the figure. Find the percentage of its stored energy dissipated after the switch S is turned to position 2.



Sol. Initially, charge on the capacitor,

$$q_i = C_i V = 2V = q$$

This charge will remain constant after switch is shifted from position 1 to position 2.

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2 \times 2} = \frac{q^2}{4}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2 \times 10} = \frac{q^2}{20}$$

$\therefore$ Energy dissipated $= U_i - U_f = \frac{q^2}{5}$

This energy dissipated $\left(= \frac{q^2}{5}\right)$ is 80% of the initial stored energy $\left(= \frac{q^2}{4}\right)$.

COMMON POTENTIAL

When two capacitors of different potentials are connected by a conducting wire, then charge flows from capacitor at higher potential to the capacitor at lower potential. This flow of charge continues till their potentials become equal, this equal potential is called **common potential.**

> Common potential, $V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

where, C_1 and C_2 are capacities of two capacitors charged to potentials V_1 and V_2, respectively.

i.e. Common potential $= \frac{\text{Total charge}}{\text{Total capacitance}}$

$\therefore$ $C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$

or $C_1 V_1 - C_1 V = C_2 V - C_2 V_2$

i.e. **Charge lost by one capacitor**

= Charge gained by the other capacitor

Note This is not true for potential, i.e. potential lost by one is not equal to potential gained by the other, as their capacities are different.

Loss of Energy in Sharing Charges

When two charged capacitors are connected to each other, they share charges, till they acquire a common potential. On sharing charges, there is always some loss of energy. However, total charge of the system remains conserved. Consider two capacitors having capacitances C_1, C_2 and potentials V_1, V_2, respectively.

Then before the two capacitors are connected together, the total energy stored in the two capacitors,

$$U = U_1 + U_2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \quad \text{...(i)}$$

When the two capacitors are connected together, total charge on the capacitor,

$$q = q_1 + q_2 = C_1 V_1 + C_2 V_2$$

Total capacitance of the two capacitors,

$$C = C_1 + C_2$$

Therefore, total energy of the two capacitors, after they are connected.

$$U' = \frac{1}{2}\cdot\frac{q^2}{C} = \frac{1}{2}\cdot\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)} \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$U - U' = \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \frac{1}{2}\cdot\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{\left[C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - (C_1V_1 + C_2V_2)^2\right]}{2(C_1 + C_2)}$$

$$= \frac{C_1C_2(V_1^2 + V_2^2 - 2V_1V_2)}{2(C_1 + C_2)}$$

$$\Rightarrow \boxed{\Delta U = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}}$$ is a positive quantity.

Since, $U - U'$ is positive, there is always a loss of energy, when two charged capacitors are connected together in the form of heat radiation due to electric current while charging.

EXAMPLE |16| A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in this process? **NCERT**

Sol. Given, $C_1 = C_2 = 600\text{pF} = 600 \times 10^{-12}\text{F}$

$$= 6 \times 10^{-10}\text{F}$$

$$V_1 = 200\text{ V}, V_2 = 0$$

$$\therefore \text{Energy lost} = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{(6\times10^{-10})^2(200-0)^2}{2\times12\times10^{-10}} = 6\times10^{-6}\text{J}$$

| TOPIC PRACTICE 2 |

OBJECTIVE Type Questions

1. The maximum electric field that a dielectric medium of a capacitor can withstand without break down (of its insulating property) is called its

(a) polarisation
(b) capacitance
(c) dielectric strength
(d) None of the above

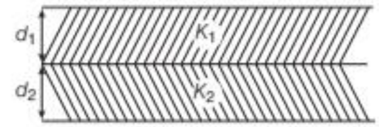
2. A parallel-plate capacitor has circular plates of radius 8 cm and plate separation 1 mm. What will be the charge on the plates if a potential difference of 100 V is applied?

(a) 1.78×10^{-8} C (b) 1.78×10^{-5} C
(c) 4.3×10^{4} C (d) 2×10^{-9} C

3. A parallel plate air capacitor has a capacitance 18 µF. If the distance between the plates is tripled and a dielectric medium is introduced, the capacitance becomes 72 µF. The dielectric constant of the medium is

(a) 4 (b) 9
(c) 12 (d) 2

4. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d(=d_1+d_2)$ and effective dielectric constant K. The K is`

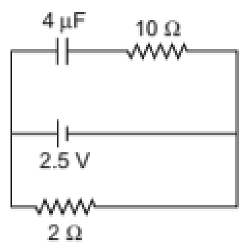


(a) $\dfrac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (b) $\dfrac{K_1d_1 + K_2d_2}{K_1 + K_2}$
(c) $\dfrac{K_1K_2(d_1 + d_2)}{(K_1d_2 + K_2d_1)}$ (d) $\dfrac{2K_1K_2}{K_1 + K_2}$

5. The capacitance of a spherical conductor is 1 µF. Its radius is

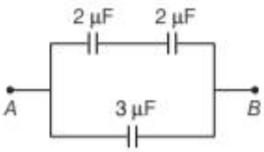
(a) 1.11 m (b) 10 m
(c) 9 km (d) 1.11 cm

6. A capacitor of 4 µF is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω. The amount of charge on the capacitor plates will be **NCERT Exemplar**



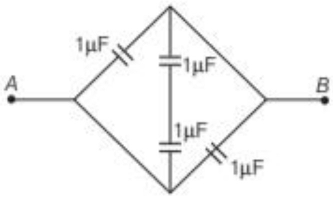
(a) 0 (b) 4 µ C
(c) 16µC (d) 8µC

7. Capacitance between points A and B is



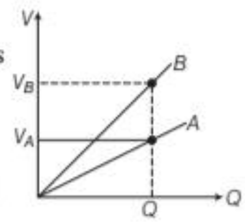
(a) 4 μF (b) $\frac{12}{7}$ μF (c) $\frac{1}{4}$ μF (d) $\frac{7}{12}$ μF

8. In the figure, the equivalent capacitance between points A and B is



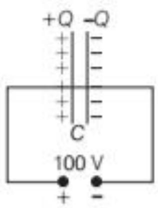
(a) 4 μF (b) 2.5 μF
(c) 2 μF (d) 0.25 μF

9. The graph shows the variation of voltage V across the plates of two capacitors A and B *versus* increase of charge Q stored in them. Which of the capacitors has higher capacitance?



(a) Capacitor A (b) Capacitor B
(c) Both (a) and (b) (d) None of these

10. A 900 pF capacitor is charged by 100 V battery in the figure. How much electrostatic energy is stored by the capacitor?



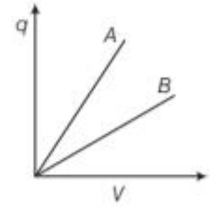
(a) 45×10^{-6} J (b) 4.5×10^{6} J
(c) 4.5×10^{-6} J (d) 0.45×10^{5} J

VERY SHORT ANSWER Type Questions

11. Distinguish between a dielectric and a conductor. **Delhi 2012**

12. Define the dielectric constant of a medium. What is its unit? **Delhi 2011**

13. The given graph shows the variation of charge q *versus* potential difference V for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that C_1. Which line (A or B) corresponds to C_1 and why? **All India 2014**



14. If the difference between the radii of the two spheres of a spherical conductor is increased, state whether the capacitance will increase or decrease.

15. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor? **Foreign 2009**

16. A spherical shell of radius b with charge Q is expanded to a radius a. Find the work done by the electrical forces in the process.

17. Distinguish between polar and non-polar dielectrics. **All India 2010 C**

18. A sensitive instrument is to be shifted from the strong electrostatic field in its environment. Suggest a possible way.

19. The safest way to protect yourself from lightning is to be inside a car. Comment. **Delhi 2009**

20. Can the potential function have a maximum or minimum in free space? **NCERT Exemplar**

21. Why does the electric conductivity of the earth's atmosphere increase with altitude?

SHORT ANSWER Type Questions

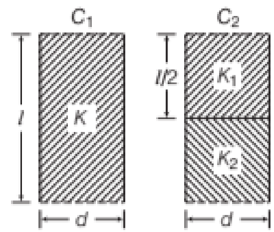
22. A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed.

State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant. **All India 2013**

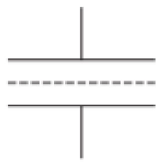
23. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor, but has the thickness $d/2$, where d is the separation between the plates.

Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. **Delhi 2013**

24. Two identical parallel plate (air) capacitors C_1 and C_2 have capacitance C each. The space between their plates is now filled with dielectrics as shown in the figure. If the two capacitors still have equal capacitance, then obtain the relation between dielectric constants K, K_1 and K_2. **Foreign 2011**



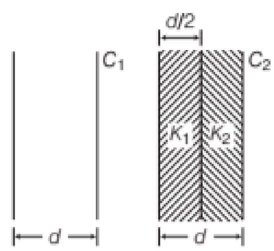
25. Figure shows a sheet of aluminium foil of negligible thickness placed between the plates of a capacitor. How will its capacitance be affected, if



(i) the foil is electrically insulated?

(ii) the foil is connected to the upper plate with a conducting wire? **Foreign 2011**

26. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in figure. Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d. **Foreign 2011**



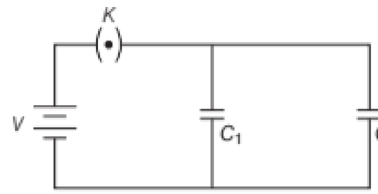
27. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. **All India 2014**

28. A parallel plate capacitor, each of plate area A and separation d between the two plates, is charged with charges $+Q$ and $-Q$ on the two plates. Deduce the expression for the energy stored in capacitor. **Foreign 2013**

29. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 = 2C_2$ are connected across a battery of V volt as shown in the figure. Initially, the key (k) is kept closed to fully charge the capacitors. The key is now thrown open and a dielectric slab of dielectric constant K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of

(i) the net capacitance and

(ii) the energies stored in the combination before and after the introduction of the dielectric slab. **Delhi 2014C**



30. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance C and having charge Q.

How will the

(i) energy stored and

(ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant K? **All India 2012**

31. Guess a possible reason, why water has a much greater dielectric constant (= 80) than mica (= 6)?

32. A 2 m insulating slab with a large aluminium sheet of area 1 m^2 on its top is fixed by a man outside his house one evening. Will he get an electric shock, if he touches the metal sheet next morning?

33. A technician has only two capacitors. By using them in series or in parallel, he is able to obtain the capacitance of 4, 5, 20 and 25 μF. What is the capacitance of both capacitors?

LONG ANSWER Type I Questions

34. (i) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?

(ii) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has thickness $\frac{1}{2}d$, where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates. **Foreign 2010**

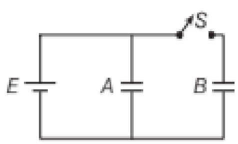
35. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain, why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions? **NCERT**

36. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2, so that the energy stored in these two cases becomes the same. **All India 2016**

37. (i) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.
(ii) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop $abcda$. **Delhi 2014**

38. A parallel plate capacitor of capacitance C is charged to a potential V by a battery. Without disconnecting the battery, the distance between the plates is tripled and a dielectric medium of $K = 10$ is introduced between the plates of the capacitor. Explain giving reasons, how will the following be affected **All India 2017**
(i) capacitance of the capacitor
(ii) charge on the capacitor and
(iii) energy density of the capacitor?

39. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S is closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **All India 2017**

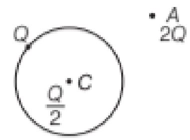


40. (i) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d.
(ii) Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting plate respectively. Find the ratio of their surface charge densities in terms of their radii. **Delhi 2014**

41. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $1/2QE$, where Q is the charge on the capacitor and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2. **NCERT**

LONG ANSWER Type II Questions

42. (i) Explain, using suitable diagram, the difference in the behaviour of a
(a) conductor and
(b) dielectric in the presence of external electric field. Define the terms polarisation of a dielectric and write its relation with susceptibility.
(ii) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $Q/2$ is placed at its centre C and an another charge $+2Q$ is placed outside the shell at a distance x from the centre as shown in figure. Find (a) the force on the charge at the centre of the shell and at point A, (b) the electric flux through the shell. **All India 2015**



43. (i) If two similar large plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, find the expression for
(a) field at points between the two plates and on outer side of the plates. Specify the direction of the field in each case.
(b) the potential difference between the plates.
(c) the capacitance of the capacitor so formed.
(ii) Two metallic spheres of radii R and $2R$ are charged, so that both of these have same surface charge density σ. If they are connected to each other with a conducting wire, in which direction will the charge flow and why? **All India 2016**

44. (i) Derive the expression for the energy stored in parallel plate capacitor. Hence, obtain the expression for the energy density of the electric field.
(ii) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than stored initially in the single capacitor. **Delhi 2015**

NUMERICAL PROBLEMS

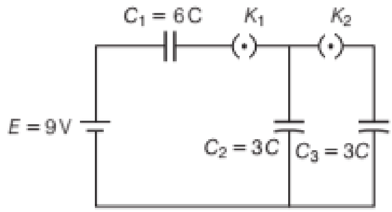
45. A capacitor of unknown capacitance is connected across a battery of V volt. The charge stored in it is $360\mu C$. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes $120\ \mu C$.

(i) Calculate the potential V and the unknown capacitance C.

(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V? **Delhi 2013**

46. In the circuit shown below, initially K_1 is closed and K_2 is opened, what are the charges on each of the capacitors? Then, K_1 was opened and K_2 was closed (order is important), what will be the charge on each capacitor now?

[Given, $C = 1\mu F$] **NCERT Exemplar**



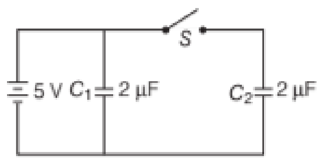
47. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of $2.5\mu C$. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(i) Determine the capacitance of the capacitor.

(ii) What is the potential of the inner sphere?

(iii) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain, why the later is much smaller. **NCERT**

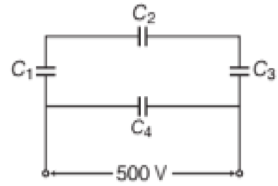
48. Figure shows two identical capacitors C_1 and C_2, each of $2\mu F$ capacitance, connected to a battery of 5 V.



Initially switch S is closed. After sometime, S is left open and dielectric slabs of dielectric constant $K = 5$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?

Delhi 2011

49. A network of four capacitors each of $12\ \mu F$ capacitance is connected to a 500 V supply as shown in the figure.



Determine

(i) the equivalent capacitance of the network and

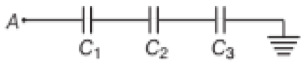
(ii) the charge on each capacitor.

All India 2012, 2010

50. Net capacitance of three identical capacitors in series is $1\mu F$. What will be their net capacitance, if connected in parallel?

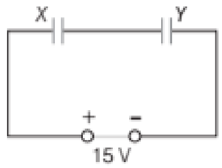
Find the ratio of energy stored in these two configurations, if they are both connected to the same source. **All India 2011**

51. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given, potential at A is 90 V, $C_1 = 20\mu F, C_2 = 30\mu F, C_3 = 15\mu F$. **Delhi 2015**



52. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. **Delhi 2017**

53. Two parallel plate capacitors X and Y have the same area of plates and same separation between them, X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.



(i) Calculate the capacitance of each capacitor, if equivalent capacitance of the combination is $4\mu F$.

(ii) Calculate the potential difference between the plates of X and Y.

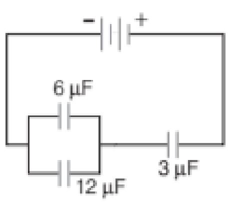
(iii) Estimate the ratio of electrostatic energy stored in X and Y. **Delhi 2016**

54. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, then determine the value of C_1 and C_2. Also, calculate the charge on each capacitor in parallel combination. **All India 2015**

55. In the following arrangement of capacitors, the energy stored in the $6\,\mu F$ capacitor is E.
Find the value of the following

(i) energy stored in $12\,\mu F$ capacitor.
(ii) energy stored in $3\,\mu F$ capacitor.
(iii) total energy drawn from the battery.

Foreign 2016



56. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. **Foreign 2012**

HINTS AND SOLUTIONS

1. (c) The maximum electric field that a dielectric medium can withstand without break down (of its insulating property) is called its dielectric strength; for air it is about $3\times 10^6\ \text{Vm}^{-1}$.

2. (a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{8.85\times 10^{-12}\times 3.14\times 0.08\times 0.08}{1\times 10^{-3}}$

$$q = CV = \frac{8.85\times 10^{-12}\times 3.14\times .08\times .08\times 100\text{ V}}{1\times 10^{-3}}$$

$$= 1.78\times 10^{-8}\text{ C}$$

3. (c) $C_0 = \dfrac{\varepsilon_0 A}{d} = 18$ ...(i)

$C = \dfrac{K\varepsilon_0 A}{3d} = 72$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{K}{3} = \frac{72}{18} = 4$$

$\therefore$ Dielectric constant, $K = 12$

4. (c) The capacitance of parallel plate capacitor filled with dielectric block has thickness d_1 and dielectric constant K_1 is given by

$$C_1 = \frac{K_1\varepsilon_0 A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric block has thickness d_2 and dielectric constant K_2 is given by

$$C_2 = \frac{K_2\varepsilon_0 A}{d_2}$$

Since, the two capacitors are in series combination, the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\text{or } C = \frac{C_1C_2}{C_1+C_2} = \frac{\dfrac{K_1\varepsilon_0 A}{d_1}\,\dfrac{K_2\varepsilon_0 A}{d_2}}{\dfrac{K_1\varepsilon_0 A}{d_1}+\dfrac{K_2\varepsilon_0 A}{d_2}} = \frac{K_1K_2\varepsilon_0 A}{K_1d_2+K_2d_1} \quad \text{...(i)}$$

But the equivalent capacitances is given by

$$C = \frac{K\varepsilon_0 A}{d_1+d_2}$$

On comparing, we have, $K = \dfrac{K_1K_2(d_1+d_2)}{K_1d_2+K_2d_1}$

5. (c) Capacitance of spherical conductor, $C = 4\pi\varepsilon_0\cdot R$

$\therefore$ Radius of conductor, $R = \dfrac{C}{4\pi\varepsilon_0} \Rightarrow C = 1\,\mu\text{F} = 1\times 10^{-6}$ F

and $\dfrac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \text{N-m/C}^2$

$\therefore$ $R = 1\times 10^{-6}\times 9\times 10^9$

$\Rightarrow$ $R = 9\times 10^3\text{m} = 9\text{ km}$

6. (d) Current flows through 2Ω resistance from left to right, is given by

$$I = \frac{V}{R+r} = \frac{2.5\text{V}}{2+0.5} = 1\text{A}$$

The potential difference across 2Ω resistance

$$V = IR = 1\times 2 = 2\text{V}$$

Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.
The charge on capacitor

$$q = CV = (4\,\mu\text{F})\times 2\text{V} = 8\,\mu\text{C}$$

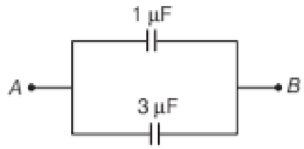
Note The potential difference across $2\,\Omega$ resistance solely occurs across capacitor as no potential drop occurs across 10Ω resistance.

7. (a) Two capacitors of $2\,\mu$F capacitance are connected in series order.
Their equivalent capacitance,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

$\therefore \quad C_S = 1\,\mu F$

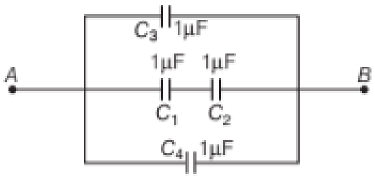
Now, $C_S = 1\,\mu F$ and $3\,\mu F$ capacitors are connected in parallel order.



Equivalent capacitance between points A and B,

$$C_{AB} = C_S + C_3 = 1 + 3 = 4\ \mu F$$

8. (b) On redrawing, the circuit is



According to the circuit, C_1 and C_2 are in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{1} = 2 \Rightarrow C = \frac{1}{2}\mu F$$

Now, C, C_3 and C_4 are in parallel order.

$$\therefore \quad C_{\text{equivalent}} = C + C_3 + C_4 = \frac{1}{2} + 1 + 1 = 2.5\,\mu F$$

9. (a) From the given graphs, find the voltages, V_A and V_B, on capacitors A and B corresponding to charge Q on each of the capacitors. Clearly,

$$V_A = \frac{Q}{C_A} \quad \text{and} \quad V_B = \frac{Q}{C_B}$$

or $$\frac{V_B}{V_A} = \frac{Q/C_B}{Q/C_A} = \frac{C_A}{C_B}$$

Since, $V_B > V_A$, $C_A > C_B$ *i.e.*, the capacitor A has the higher capacitance.

10. (c) The charge on the capacitor is

$$q = CV = 900 \times 10^{-12}\ F \times 100\ V = 9 \times 10^{-8} C$$

The energy stored by the capacitor is

$$= (1/2)\,CV^2 = (1/2)\,qV$$

$$= 1/2 \times 9 \times 10^{-8} C \times 100\ V = 4.5 \times 10^{-6}\ J$$

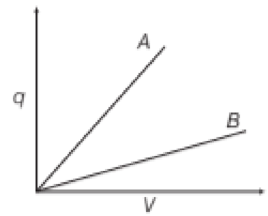
11. Dielectrics are non-conductors and do not have free electrons at all. While conductor has free electrons which makes it able to pass the electricity through it.

12. When a dielectric slab is introduced between the plates of charged capacitor or in the region of electric field, an electric field E_p induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence, net electric field inside the dielectric get reduced to $E_0 - E_p$, where E_0 is external electric field.

The ratio of applied external electric field and reduced electric field is known as dielectric constant K of dielectric medium,

i.e. $K = \dfrac{E_0}{E_0 - E_p}$ and it is a dimensionless quantity.

13. Line B corresponds to C_1 because slope (q/V) of B is less than slope of A.



14. Capacitance of a spherical capacitor, $C = \dfrac{4\pi\varepsilon_0 K r_1 r_2}{r_1 - r_2}$

$\Rightarrow C \propto \dfrac{1}{r_1 - r_2}$. If $r_1 - r_2$ is increased, C decreases.

15. If a metal plate is introduced between the plates of a charged parallel plate capacitor, then capacitance of parallel plate capacitor will become infinite.

16. Work done by electrical forces in the process

= Final stored energy – Initial stored energy

$$= \frac{Q^2}{2C_2} - \frac{Q^2}{2C_1} = \frac{Q^2}{2(4\pi\varepsilon_0 a)} - \frac{1}{2}\cdot\frac{Q^2}{(4\pi\varepsilon_0 b)} = \frac{Q^2}{8\pi g\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

17. **Polar dielectrics**

A polar dielectric has permanent electric dipole moment (**p**) in absence of electric field.

Non-polar dielectrics

A non-polar dielectric having zero dipole moment in its normal state.

18. For this, the instrument must be enclosed fully in a metallic cover. This will provide an electrostatic shielding to the instrument.

19. The body of the car is metallic. It provides electrostatic shielding to the person in the car, because electric field inside the car is zero. The discharging due to lightning passes to the ground through the metallic body of the car.

20. No, the absence of atmosphere around conductor prevents the phenomenon of electric discharge or potential leakage and hence, potential function do not have a maximum or minimum in free space.

21. This is because of ionisation caused by highly energetic cosmic ray particles from cosmos, which are hitting the atmosphere of the earth.

22. The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is given by, $C = \dfrac{K\varepsilon_0 A}{d}$.

The capacitance of the parallel plate capacitor decreases with the removal of dielectric medium as for air or vacuum $K = 1$. After disconnection from battery, charge stored will remain the same due to conservation of charge. The energy stored in an isolated charge capacitor $= \frac{q^2}{2C}$.

As q is constant, energy stored $\propto 1/C$. C decrease with the removal of dielectric medium, therefore energy stored increases. Since, q is constant and $V = q/C$ and C decreases which in turn increases V and therefore E increases $E = V/d$.

23. Initially, when there is a vacuum between two plates, the capacitance of the plate is $C_0 = \frac{\varepsilon_0 A}{d}$, where A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced. Now, if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now, the gap between plates is divided in two parts, for distance t, there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0.

If V be the potential difference between the plates of the capacitor, then $V = Et + E_0(d - t)$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} = \frac{d}{2}(E + E_0) \qquad \left[\because t = \frac{d}{2}\right]$$

$$\Rightarrow \quad V = \frac{d}{2}\left(\frac{E_0}{K} + E_0\right) = \frac{dE_0}{2K}(K+1) \qquad \left[\text{as, } \frac{E_0}{E} = K\right]$$

Now, $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A} \Rightarrow V = \frac{d}{2K} \cdot \frac{q}{\varepsilon_0 A}(K+1)$

We know that, $C = \frac{q}{V} = \frac{2K\varepsilon_0 A}{d(K+1)}$

24. After inserting the dielectric medium, let their capacitances become C_1' and C_2'.

For C_1 $\quad C_1' = KC \qquad$ ...(i)

For C_2 $\quad C_2' = \frac{K_1 \varepsilon_0 (A/2)}{d} + \frac{K_2 \varepsilon_0 (A/2)}{d}$

C_2 acts as if two capacitors each of area $A/2$ and separation d are connected in parallel combination

$$C_2' = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2}{2}\right)$$

$$C_2' = C\left(\frac{K_1 + K_2}{2}\right) \left[\because C = \frac{\varepsilon_0 A}{d}\right] \qquad \text{...(ii)}$$

According to the problem, $C_1' = C_2'$

$$\Rightarrow \quad KC = C\left(\frac{K_1 + K_2}{2}\right)$$

$$\Rightarrow \quad K = \frac{K_1 + K_2}{2}$$

25. (i) The system will be equivalent to two identical capacitors connected in series combination in which two plates of each capacitor have separation half of the original separation.

Thus, new capacitance of each capacitor

$$C' = 2C \qquad \left[\because C \propto \frac{1}{d}\right]$$

$\because$ C and C' are in series.

$$\Rightarrow \quad C_{net} = \frac{2C \times 2C}{2C + 2C} = C$$

$$C_{net} = C \qquad \text{[original capacitor]}$$

(ii) System reduces to a capacitor whose separation reduces to half of original one.

$\therefore$ New capacitance, $C' = 2C$

26. After introducing the dielectric medium of dielectric constants K_1 and K_2, capacitor acts as if it consists of two capacitors, each having plates of area A and separation $\frac{d}{2}$ connected in series combination for

$$C_1 = \frac{\varepsilon_0 A}{d} \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{\left(\frac{K_1 \varepsilon_0 A}{d/2}\right)} + \frac{1}{\left(\frac{K_2 \varepsilon_0 A}{d/2}\right)}$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d}\right)}\left(\frac{1}{2K_1} + \frac{1}{2K_2}\right)$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{2C_1}\left(\frac{K_2 + K_1}{K_1 K_2}\right)$$

$$\Rightarrow \quad C_2 = C_1\left(\frac{2K_1 K_2}{K_1 + K_2}\right)$$

The capacitors will be in series.

27. Let q be the charge on the charged capacitor.

$\therefore$ Energy stored in it is given by $U = \frac{q^2}{2C}$

When another uncharged similar capacitor is connected, then the net capacitance of the system is given by

$$C' = 2C$$

The charge on the system remains constant. So, the energy stored in the system is given by

$$U' = \frac{q^2}{2C'} = \frac{q^2}{4C} \qquad [\because C' = 2C]$$

Thus, the required ratio is given by $\frac{U'}{U} = \frac{q^2/4C}{q^2/2C} = \frac{1}{2}$

28. Refer to text on page 92.

29. (i) Given, $\quad C_1 = 2C_2 \qquad$ ...(i)

Net capacitance before filling the gap with dielectric slab is given by

$$C_{initial} = C_1 + C_2 \qquad \text{[from Eq. (i)]}$$

$$C_{initial} = 2C_2 + C_2 = 3C_2 \qquad \text{...(ii)}$$

Net capacitance after filling the gap with dielectric slab of electric constant K

$C_{\text{initial}} = KC_1 + KC_2 = K(C_1 + C_2)$ [from Eq. (ii)]

$C_{\text{final}} = 3KC_2$ ...(iii)

Ratio of net capacitance is given by

$$\frac{C_{\text{initial}}}{C_{\text{final}}} = \frac{3C_2}{3KC_2} = \frac{1}{K}$$ [from Eqs. (ii) and (iii)]

(ii) Energy stored in the combination before introducing the dielectric slab,

$$U_{\text{initial}} = \frac{Q^2}{3C_2}$$ ...(iv)

Energy stored in the combination after introducing the dielectric slab,

$$U_{\text{final}} = \frac{Q^2}{3KC_2}$$ ... (v)

Ratio of energies stored

$$\frac{U_{\text{initial}}}{U_{\text{final}}} = \frac{K}{1}$$ [from Eqs. (iv) and (v)]

30. (i) Refer to text on page 92.

(ii) Refer to text on pages 88 and 92.

31. Dielectric constant of water is much greater than that of mica because of the following reasons

(i) water molecules have a symmetrical shape as compared to mica

(ii) water molecules have permanent dipole moment.

32. Yes, the man will get an electric shock, if he touches the metal slab next morning because the steady discharging current in the atmosphere charges up the aluminium sheet. As a result, its voltage rises gradually. The rise in voltage depends on the capacitance of the capacitor formed by aluminium slab and ground.

33. Let the two capacitors be C_1 and C_2, capacitance will be maximum when connected in parallel.

i.e. $C_1 + C_2 = 25$

Capacitance will be minimum when connected in series.

i.e. $$\frac{C_1C_2}{C_1 + C_2} = 4$$

Since, we are left with only two values 5 μF and 20 μF.

So, the value of capacitances will be 5 μF and 20 μF.

34. (i) Refer to text on page 88.

(ii) The thickness of dielectric slab is $\frac{d}{2}$, i.e.

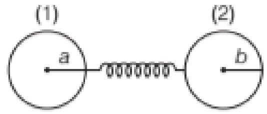
$$t = \frac{d}{2}$$

The capacitance of a capacitor due to dielectric slab is

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$$

$$= \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{2\varepsilon_0 A}{d\left(1 + \frac{1}{k}\right)}$$

35. As the two conducting spheres are connected to each other by a wire, the charge always flows from higher potential to lower potential till both have same potential.



Capacitance of sphere (1), $C_1 = 4\pi\varepsilon_0 a$

Capacitance of sphere (2), $C_2 = 4\pi\varepsilon_0 b$

Then, Charge Q_1 on C_1, $Q_1 = C_1V$ ...(i)

and Charge Q_2 on C_2, $Q_2 = C_2V$ ...(ii)

where, V is the same potential on both the spheres.

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$ [from Eqs. (i) and (ii)]

Putting the values of C_1 and C_2, we get

$$\frac{Q_1}{Q_2} = \frac{4\pi\varepsilon_0 a}{4\pi\varepsilon_0 b} = \frac{a}{b} \Rightarrow \frac{Q_1}{Q_2} = \frac{a}{b}$$ ...(iii)

Charge density on sphere (1), $\sigma_1 = \frac{\text{Charge}}{\text{Surface area}} = \frac{Q_1}{4\pi a^2}$

Charge density on sphere (2), $\sigma_2 = \frac{Q_2}{4\pi b^2}$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{b^2}{a^2}\cdot\frac{Q_1}{Q_2} = \frac{b^2}{a^2}\cdot\frac{a}{b}$$ [from Eq. (iii)]

or $$\frac{\sigma_1}{\sigma_2} = \frac{b}{a}$$ ...(iv)

The ratio of electric field on both spheres.

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$$ [from Eq. (iv)]

As, charge density is inversely proportional to radius. Thus, for flatter portions, the radius is more and at pointed ends, radius is less, so the charge density is more at pointed or sharp ends.

36. Total energy stored in series or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors. In parallel combination, energy stored in the capacitor

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_1^2$$ ...(i)

In series combination, energy stored in the capacitor

$$= \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}V_2^2$$ ...(ii)

According to the question, energy in both the cases is same so,

$$\left(\frac{1}{2}C_1 + \frac{1}{2}C_2\right)V_1^2 = \frac{C_1C_2}{2(C_1 + C_2)}V_2^2$$

$$\Rightarrow \quad \frac{V_1^2}{V_2^2} = \frac{C_1C_2 \times 2}{2(C_1 + C_2)(C_1 + C_2)}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{\sqrt{C_1C_2}}{C_1 + C_2}$$

But $\quad \dfrac{C_1}{C_2} = \dfrac{1}{2}$

$\Rightarrow \quad C_2 = 2C_1$

So, $\quad \dfrac{V_1}{V_2} = \dfrac{\sqrt{C_1 \times 2C_1}}{C_1 + 2C_1} = \dfrac{\sqrt{2}C_1}{3C_1} = \dfrac{\sqrt{2}}{3}$

37. (i) Refer to text on page 92.

(ii) Due to conservative nature of electric force, the work done in moving a charge in a close path in a uniform electric field is zero.

38. On introducing the dielectric slab to fill the gap between plates of capacitor completely when capacitor is connected with battery.

(i) The capacitance of capacitor becomes K times of original capacitor.

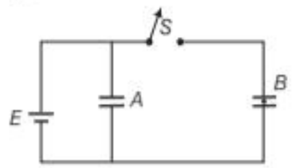
$$C' = KC = 10C$$

(ii) The potential difference V between capacitors is same due to connectivity with battery and hence, charge q' becomes K times of original charge as

$$q' = C'V' = (KC)(V) = K(CV) = Kq = 10CV$$

(iii) Refer to text on page 92.

39. The given figure is shown below.



When switch S is closed, the potential difference across capacitors A and B are same

i.e. $\quad V = \dfrac{Q_A}{C} = \dfrac{Q_B}{C}$

Initial charges on capacitors

$$Q_A = Q_B = CV$$

When the dielectric is introduced, the new capacitance of either capacitor

$$C' = KC$$

As switch S is opened, the potential difference across capacitor A remains same (V volts).

Let potential difference across capacitor B be V'. When dielectric is introduced with switch S open (i.e. battery disconnected), the charges on capacitor B remains unchanged, so

$$Q_B = CV = C'V'$$

$\Rightarrow \quad V' = \dfrac{C}{C'}V = \dfrac{V}{K}$ volt

Initial energy of both capacitors

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Final energy of both capacitors

$$U_f = \frac{1}{2}C'V^2 + \frac{1}{2}C'V'^2$$

$$= \frac{1}{2}(KC)V^2 + \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2$$

$$= \frac{1}{2}CV^2\left[K + \frac{1}{K}\right]$$

$$= \frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)$$

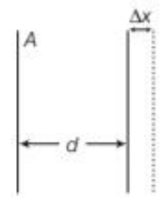
$\Rightarrow \quad \dfrac{U_i}{U_f} = \dfrac{CV^2}{\frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)} = \dfrac{2K}{K^2+1}$

40. (i) Refer to text on page 87.

(ii) $\dfrac{\sigma_1}{\sigma_2} = \dfrac{R_2}{R_1}$;

Here, we can use the concept that the work done in displacing the plates against the force is equal to the increase in energy of the capacitor.

41. Let the distance between the plates be increased by a very small distance Δx. The force on each plate is F. The amount of work done in increasing the separation by Δx, i.e.



$$W = F \cdot \Delta x \qquad \text{...(i)}$$

Increase in volume of capacitor

$= \text{Area of plates} \times \text{Increased distance}$

$= A \cdot \Delta x$

$$u = \text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

Increase in energy $= u \times \text{volume} = u \cdot A \cdot \Delta x$ ...(ii)

As, energy = work done (W)

$\Rightarrow \quad F \cdot \Delta x = u \cdot A \cdot \Delta x$ [from Eqs. (i) and (ii)]

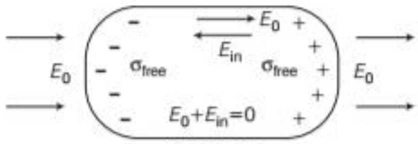
$\Rightarrow \quad F = u \cdot A$

$= \dfrac{1}{2}\varepsilon_0 E^2 \cdot A \qquad \left[\because u = \dfrac{1}{2}\varepsilon_0 E^2 \text{ and } E = \dfrac{V}{d}\right]$

$= \dfrac{1}{2}\varepsilon_0 \cdot \dfrac{V^2}{d^2} \cdot A = \left(\dfrac{\varepsilon_0 A}{d} \cdot V\right)\dfrac{V}{d} \times \dfrac{1}{2}$

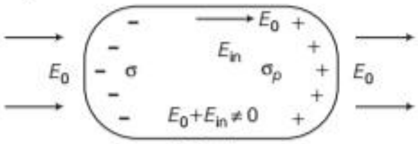
$= \dfrac{1}{2} \cdot E \cdot C \cdot V = \dfrac{1}{2}QE \qquad \left[\because C = \dfrac{\varepsilon_0 A}{d}, CV = Q\right]$

42. (i) (a) When a capacitor is placed in an external electric field, the free charges present inside the conductor redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved, i.e. when the two fields cancel each other and the net electrostatic field in the conductor becomes zero.



Conductor

(b) In contrast to conductors, dielectrics are non-conducting substances, i.e. they have no charge carriers. Thus, in a dielectric, free movement of charges in not possible. It turns out that the external field induces dipole moment by stretching molecules of the dielectric. The collective effect of all the molecular dipole moments is the net charge on the surface of the dielectric which produces a field that opposes the external field. However, the opposing field is so induced, that does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of dielectric.



Dielectric

Both polar and non-polar dielectrics develop net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and is denoted by P for linear isotropic dielectrics.

$$P = \chi E$$

where, χ is constant of proportionality and is called electric susceptibility of the electric slab.

(ii) (a) At point C, inside the shell, electric field inside a spherical shell is zero.

Thus, the force experienced by charge at centre C will also be zero.

$\because \quad F_C = qE \; (E_{\text{inside}} \text{ the shell} = 0)$

$\therefore \quad F_C = 0$

At point A, $|F_A| = 2Q\left[\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{3Q/2}{x^2}\right]$

$$F = \frac{3Q^2}{4\pi\varepsilon_0 x^2}, \text{ away from shell.}$$

(b) Electric flux through the shell,

$$\phi = \frac{1}{\varepsilon_0} \times \text{magnitude of charge enclosed by shell}$$

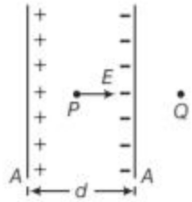
$$= \frac{1}{\varepsilon_0} \times \frac{Q}{2} = \frac{Q}{2\varepsilon_0}$$

43. (i) According to the question,

(a) Electric field due to a plate of positive charge at point

$$P = \frac{\sigma}{2\varepsilon_0}$$

Electric field due to other plate $= \dfrac{\sigma}{2\varepsilon_0}$



Since, they have same direction, so

$$E_{\text{net}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

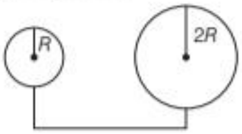
Outside the plate, electric field will be zero because of opposite direction.

(b) Potential difference between the plates is given by $V = Ed = \dfrac{\sigma d}{\varepsilon_0}$ $\quad \left[\because E = \dfrac{\sigma}{\varepsilon_0}\right]$

(c) Capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d}\varepsilon_0 = \frac{\varepsilon_0 A}{d}$$

(ii) According to question,



Potential at the surface of radius R,

$$V = \frac{kq}{R} \qquad [\because q = \sigma \times 4R^2]$$

$$= \frac{k\sigma 4\pi R^2}{R} = \sigma k 4\pi R = 4k\sigma\pi R$$

Potential at the surface of radius $2R$,

$$V' = \frac{kq}{2R} \qquad [\because q = \sigma \times 4\pi(2R)^2 = 16\sigma\pi R^2]$$

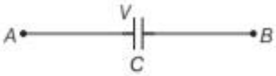
So, $\quad V' = \dfrac{k\sigma 16\pi R^2}{2R} = 8k\sigma\pi R$

Since, the potential of bigger sphere is more. So, charge will flow from sphere of radius $2R$ to sphere of radius R.

44. (i) Refer to text on page 92.

(ii) Initially, if we consider a charged capacitor, then its charge would be

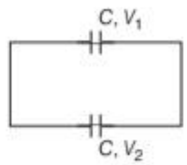
$$Q = CV$$



and energy stored, $U_1 = \dfrac{1}{2}CV^2$ ...(i)

Then, this charged capacitor is connected to uncharged capacitor.

Let the common potential be V_1. The charge flows from first capacitor to the other capacitor unless both the capacitors attain common potential



$Q_1 = CV_1$ and $Q = CV_2$

Applying conservation of charge, $Q = Q_1 + Q_2$

$\Rightarrow \quad CV = CV_1 + CV_2$

$\Rightarrow \quad V = V_1 + V_2 \Rightarrow V_1 = \dfrac{V}{2}$

Total energy stored, $U_2 = \dfrac{1}{2}CV_1^2 + \dfrac{1}{2}CV_2^2$

$$= \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2 \Rightarrow U_2 = \frac{1}{4}CV^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$U_2 < U_1$$

Hence, energy stored in the combination is less than that stored initially in single capacitor.

45. (i) We have initial voltage, $V_1 = V$ volt
and charge stored, $Q_1 = 360\mu C$

$$Q_1 = CV_1 \quad \text{...(i)}$$

Charged potential, $V_2 = V - 120$

$$Q_2 = 120\,\mu C$$

$$\Rightarrow \quad Q_2 = CV_2 \quad \text{...(ii)}$$

By dividing Eq. (ii) from Eq. (i), we get

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2} \Rightarrow \frac{360}{120} = \frac{V}{V-120}$$

$\Rightarrow \quad V = 180$ V

$$\therefore \quad C = \frac{Q_1}{V_1} = \frac{360\times10^{-6}}{180} = 2\times10^{-6}\text{ F}$$

$= 2\,\mu F$

Hence, the potential, $V = 180$ V and unknown capacitance is $2\,\mu F$.

(ii) If the voltage applied had increased by 120 V, then $V_3 = 180 + 120 = 300$ V
Hence, charge stored in the capacitor,
$Q_3 = CV_3 = 2\times10^{-6}\times300 = 600\,\mu C$

46. In the circuit, when initially K_1 is closed and K_1 is opened, the capacitor C_1 and C_2 acquire potential difference V_1 and V_2, respectively. So, we have

$$V_1 + V_2 = E$$

and $\quad V_1 + V_2 = 9V$

Also, in series combination, $V \propto 1/C$

$$V_1 : V_2 = 1/6 : 1/3$$

On solving,

$\Rightarrow \quad V_1 = 3V$ and $V_2 = 6V$

$\therefore \quad Q_1 = C_1V_1 = 6\,\mu C \times 3V$

$= 18\,\mu C \quad [\because C = 1\,\mu F]$

$\Rightarrow \quad Q_2 = C_2V_2 = 3\,\mu C \times 6V = 18\,\mu C$

and $\quad Q_3 = 0$

When K_1 was opened and K_2 was closed, the parallel combination of C_2 and C_3 in series with C_1.
[Charge on C_1 remains unchanged]

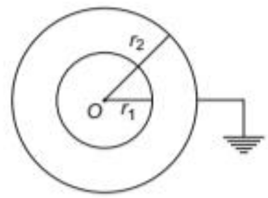
i.e. $\quad Q_1' = Q_2 = 18\,\mu C$

Charge on C_2 is shared between C_2 and C_3 in parallel.

As, $\quad C_2 = C_3$

$$\therefore \quad Q_2' = Q_2 = \frac{Q_2}{2} = \frac{18}{2} = 9\,\mu C \quad [\because Q_2 = 18\,\mu C]$$

47. Radius of inner sphere, $r_1 = 12$ cm
Radius of outer sphere, $r_2 = 13$ cm
and charge on inner sphere, $q = 2.5\,\mu C$
The dielectric constant, $K = 32$



(i) Capacitance of a spherical capacitor,

$$C = \frac{4\pi\varepsilon_0 K r_1 r_2}{r_1 - r_2} = \frac{1}{9\times10^9}\cdot\frac{32\times12\times13\times10^{-4}}{(13-12)\times10^{-2}}$$

$= 5.5\times10^{-9}$ F

(ii) Electric potential of inner sphere,

$= 4.5\times10^2$ V

(iii) Capacitance of an isolated sphere of radius, $r = 12$ cm

$$C = 4\pi\varepsilon_0 r = \frac{1}{9\times10^9}\times12\times10^{-2}$$

$= 1.33\times10^{-11}$ F

The capacitance of an isolated sphere is much smaller as compared to the spherical capacitor because the outer sphere is earthed. The potential difference decreases and hence the capacitance increases.

48. Two identical capacitors C_1 and C_2 get fully charged with 5 V battery initially.
So, the charge and potential difference on both capacitors becomes

$$q = CV$$

$$= 2\times10^{-6}\times5\text{ V} = 10\,\mu C$$

and $\quad V = 5$ V

On introduction of dielectric medium of $K = 5$.

For C_1 (Continue to be connected with battery)

Potential difference of C_1, $V' = 5$ V

Capacitance, $C_1' = KC = 5\times2 = 10\,\mu F$

Charge, $q' = C'V' = 10\times5 = 50\,\mu C$

For C_2 (Disconnected from battery)

Charge, $q' = q = 10\,\mu C$

$\therefore$ Potential difference, $V' = \dfrac{V}{K} = \dfrac{5}{5} = 1$ V

49. (i) Here, C_1, C_2 and C_3 are in series, therefore, their equivalent capacitance

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad C' = \frac{C}{3} = \frac{12}{3} = 4\ \mu\text{F}$$

Now, C' and C are in parallel combination.

$$\therefore \quad C_{\text{net}} = C' + C$$
$$= 4\ \mu\text{F} + 12\ \mu\text{F} = 16\ \mu\text{F}$$

(ii) Being C' and C are in parallel, 500 V potential difference is applied across them.

$\therefore$ Charge on C'

$$q_1 = C'V = (4\ \mu\text{F}) \times 500 = 2000\ \mu\text{C}$$

$\therefore$ C_1, C_2 and C_3 capacitors each will have $2000\ \mu\text{C}$ charge.

$\therefore$ Charge on C_4, $q_2 = C \times V$

$$= 12 \times 500 = 6000\ \mu\text{C}$$

50. If n identical capacitors, each of capacitance C are connected in series combination give equivalent capacitance, $C_s = \frac{C}{n}$ and when connected in parallel combination, then equivalent capacitance, $C_p = nC$.

Also, for same voltage, energy stored in the capacitor is given by

$$U = \frac{1}{2}CV^2 \qquad [\text{for } V = \text{constant}]$$

$$\Rightarrow \quad U \propto C$$

In series combination, $C_s = \frac{C}{n}$

$$\Rightarrow \quad C_s = 1\ \mu\text{F} \qquad [\because n = 3]$$

$$\Rightarrow \quad C = nC_s = 3 \times 1\ \mu\text{F} = 3\ \mu\text{F}$$

In parallel combination, $C_p = nC = 3 \times 3 = 9\ \mu\text{F}$

For same voltage, $U \propto C$

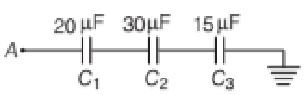
$$\Rightarrow \quad \frac{U_s}{U_p} = \frac{C_s}{C_p}$$

$$\Rightarrow \quad \frac{U_s}{U_p} = \frac{C/n}{nC} = \frac{1}{n^2}$$

$$\Rightarrow \quad \frac{U_s}{U_p} = \frac{1}{(3)^2} = \frac{1}{9}$$

or $\quad U_s : U_p = 1 : 9$

51. Consider the given figure,



Given, $\quad C_1 = 20\ \mu\text{F}$, $C_2 = 30\ \mu\text{F}$, $C_3 = 15\ \mu\text{F}$

Potential at $A = 90$ V

As, we can see that capacitor C_3 is earthed, therefore, potential across C_3 will be zero.

Since, capacitors C_1, C_2 and C_3 are connected in series, therefore

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$= \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{3+2+4}{60} = \frac{9}{60}$$

$$\Rightarrow \quad C_{eq} = \frac{60}{9} = \frac{20}{3}\ \mu\text{F}$$

Since, charge remains same in series combination.

So, $\quad Q = C_{eq}V = \frac{20}{3} \times 90$

$$\Rightarrow \quad Q = 600\ \mu\text{C}$$
$$= 600 \times 10^{-6}\ \text{C}$$
$$= 6 \times 10^{-4}\ \text{C}$$

$\therefore$ Potential difference across $C_2 = \frac{Q}{V_2}$

$$\Rightarrow \quad V_2 = \frac{Q}{C_2}$$

$$\Rightarrow \quad V_2 = \frac{6 \times 10^{-4}}{30 \times 10^{-6}} = 20\ \text{V}$$

$\therefore$ Energy stored in capacitor C_2 is given by

$$E = \frac{1}{2}C_2V_2^2$$
$$= \frac{1}{2} \times 30 \times 10^{-6} \times (20)^2$$
$$= \frac{1}{2} \times 30 \times 400 \times 10^{-6}\ E$$
$$= 6 \times 10^{-3}\ \text{J}$$

52. Energy stored in capacitor $= \frac{1}{2}C_1V^2$

$$= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = 15 \times 10^{-9}\ \text{J}$$

With other capacitor 6 pF in series.

Total capacitance (C)

$$= \frac{C_1 \times C_2}{C_1 + C_2} = \frac{6 \times 12}{6 + 12} = \frac{12 \times 6}{18} = 4\ \text{pF}$$

Charge stored in each capacitor is same and is given by

$$Q = CV$$
$$= 4 \times 10^{-12} \times 50\ \text{C} = 2 \times 10^{-10}\ \text{C}$$

Each of the capacitors will have charge equal to Q

$$= 2 \times 10^{-10}\ \text{C}$$

Potential on capacitors with capacitance 12 pF is

$$= \frac{Q}{C_1} = \frac{2 \times 10^{-10}}{12 \times 10^{-12}}\ \text{V} = 16.67\ \text{V}$$

Potential on capacitor with capacitance 6 pF is

$$= \frac{2 \times 10^{-10}}{6 \times 10^{-12}}\ \text{V} = 33.33\ \text{V}$$

53. According to question, let the capacitance of X be C, so capacitance of $Y = \varepsilon_r C = 4C$ $[\because \varepsilon_r = 4]$

(i) Equivalent capacitance $= \frac{C \times 4C}{C + 4C}$

$[\because X$ and Y are in series$]$

$= \frac{4C^2}{5C} = \frac{4C}{5}$ and it is given that $\frac{4C}{5} = 4\mu F$

So, $4C = 20\mu F =$ capacitance of Y

Capacitance of $X = C = \frac{20}{4} = 5\,\mu F$

(ii) Charge flowing through the capacitor is given by

$$q = CV = \frac{4C}{5} \times 15 = \frac{4 \times 5}{5} \times 15 = 60\,\mu C$$

Now, let the potential difference between plates of capacitors X and Y are V_x and V_y, respectively.

So, $V_x = \frac{q}{C_x} = \frac{60}{5} = 12\,V$

and $V_y = \frac{q}{C_y} = \frac{60}{20} = 3\,V$

(iii) Electrostatic energy stored in capacitance

$$X(E_x) = \frac{1}{2}CV_x^2 \quad \text{...(i)}$$

Similarly for Y, $E_y = \frac{1}{2}4CV_y^2$ ...(ii)

From Eqs. (i) and (ii), we get

$$\text{Ratio} = \frac{E_x}{E_y} = \frac{\frac{1}{2}CV_x^2}{\frac{1}{2}4CV_y^2} = \frac{V_x^2}{4V_y^2} = \frac{12 \times 12}{4 \times 3 \times 3} = 4:1$$

54. When the capacitors are connected in parallel, equivalent capacitance, $C_P = C_1 + C_2$.

The energy stored in the combination of the capacitors,

$$E_P = \frac{1}{2}C_P V^2 = \frac{1}{2}(C_1 + C_2)(100)^2 = 0.25\,J$$

$$\Rightarrow \quad C_1 + C_2 = 5 \times 10^{-5} \quad \text{...(i)}$$

When the capacitors are connected in series, equivalent capacitance,

$$C_S = \frac{C_1 C_2}{C_1 + C_2}$$

The energy stored in the combination of the capacitors,

$$E_S = \frac{1}{2}C_S V^2$$

$$\Rightarrow \quad E_S = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(100)^2$$

$$= \frac{1}{2} \times \frac{C_1 C_2}{5 \times 10^{-5}}(100)^2 = 0.45\,J$$

$$\Rightarrow \quad C_1 C_2 = 0.045 \times 10^{-4} \times 5 \times 10^{-5} \times 2$$

$$= 4.5 \times 10^{-10}$$

$$\because \quad (C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2$$

$$\Rightarrow \quad (C_1 - C_2)^2 = 25 \times 10^{-10} - 4 \times 4.5 \times 10^{-10}$$

$$= 7 \times 10^{-10}$$

$$\Rightarrow \quad (C_1 - C_2) = \sqrt{7 \times 10^{-10}} = 2.64 \times 10^{-5}$$

$$\Rightarrow \quad C_1 - C_2 = 2.64 \times 10^{-5} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$C_1 = 35\,\mu F$ and $C_2 = 15\,\mu F$

$$\Rightarrow \quad Q_1 = C_1 V = 35 \times 10^{-6} \times 100$$

$$= 35 \times 10^{-4}\,C$$

and $$Q_2 = C_2 V = 15 \times 10^{-6} \times 100$$

$$= 15 \times 10^{-4}\,C$$

55. (i) As given in the question, energy of the $6\,\mu F$ capacitor is E. Let V be the potential difference along the capacitor of capacitance $6\,\mu F$.

Now, $\frac{1}{2}CV^2 = E$

$$\frac{1}{2} \times 6 \times 10^{-6} \times V^2 = E$$

$$\Rightarrow \quad V^2 = \frac{E}{3} \times 10^6 \quad \text{...(i)}$$

Since, potential is same for parallel connection, the potential through $12\,\mu F$ capacitor is also V. Hence, energy of $12\,\mu F$ capacitor is

$$E_{12} = \frac{1}{2} \times 12 \times 10^{-6} \times V^2 \quad \text{[from Eq. (i)]}$$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times \frac{E}{3} \times 10^6 = 2E$$

(ii) Since, charge remains constant in series, the charge on $6\mu F$ and $12\,\mu F$ capacitors combined will be equal to the charge on $3\,\mu F$ capacitor.

Using the formula, $Q = CV$, we can write

$$\Rightarrow (6 + 12) \times 10^{-6} \times V = 3 \times 10^{-6} \times V'$$

$$V' = 6\,V$$

Squaring on both sides, we get

$$V'^2 = 36V^2$$

Putting the value of V^2 from Eq. (i), we get

$$V'^2 = 36 \times \frac{E}{3} \times 10^6$$

$$\Rightarrow \quad V'^2 = 12E \times 10^6$$

$$\therefore \quad E_3 = \frac{1}{2} \times 3 \times 10^{-6} \times 12E \times 10^6$$

$$= 18E$$

(iii) Total energy drawn from battery is

$$E_{\text{total}} = E + E_{12} + E_3$$

$$= E + 2E + 18E$$

$$= 21E$$

56. 3×10^{-6} J; refer to Example 16 on page 94.

SUMMARY

- **Electrostatic Potential** It is the amount of work done (w) in moving a unit positive test charge (q) without acceleration form infinity to that point against the electrostatic force

$$\therefore \quad V = \frac{W}{q}$$

Its SI unit is volt (V) and $1\,V = 1\,J/C$.

- **Electrostatic Potential Difference** Electrostatic potential difference between two points P and Q is equal to the work done (W_{QP}) by external force in moving a unit positive charge (q_0) against the electrostatic force from point Q to P along any path between these two points

$$\therefore \quad \Delta V = \frac{W_{QP}}{q_0}$$

Its SI unit is volt and $1V = 1\,JC^{-1}$.

Electric Potential due to a Point Charge

It can be given as, $V = \dfrac{q}{4\pi\varepsilon_0 r}$

Here, r is distance of the point from the charge.

Electrostatic potential at any point P due to a system of n point charges $q_1, q_2, \ldots, q_n$ whose position vectors are $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ respectively, is given by

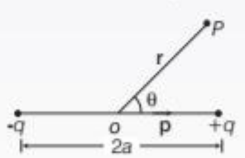
$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|}$$

where, $\mathbf{r}$ is the position vector of point P w.r.t. the origin.

- Electrostatic potential due to a thin charged spherical shell carrying charge q and radius R respectively, at any point P lying
  - (i) inside the shell is $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$
  - (ii) on the surface of shell is $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$
  - (iii) outside the shell is $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r}$ for $r > R$

    where, r is the distance of point P from the centre of the shell.

- Electrostatic potential due to an electric dipole at any point P whose position vector is $\mathbf{r}$ w.r.t. mid-point of dipole is given by



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2} \text{ or } V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{|r|^2}$$

where, θ is the angle between $\hat{\mathbf{r}}$ and $\mathbf{p}$.

- **Equipotential Surfaces** Any surface which has same electrostatic potential at every point is called an equipontential surface.
- **Equipotential Surface in Different Cases** The equipotential surface can be obtained for different cases:
  - (i) For a point charge, it is spherical surface.
  - (ii) For a uniform electric field, it is plane surface.
- **Relation between Electric Field and Electrostatic Potential**

  It can be given as, $E = -\dfrac{\partial V}{\partial r} = -$ (Potential gradient)

  where, negative sign indicates that the direction of electric field is from higher potential to lower potential, i.e. in the direction of decreasing potential.

- **Electrostatic Potential Energy of a System of Charges** It is defined as, the total work done in bringing the different charges to their respective positions from infinitely large mutual separations.
- **Due to System of Two Point Charges** It can be given by,

$$U = W = \frac{kq_1q_2}{r_{12}}$$

- **Due to System of Three Point Charges** It can be given by,

$$U = \left[K \sum_{i=1}^{3} \sum_{\substack{j=1 \\ j \neq 1}}^{3} \frac{q_i q_j}{r_{ij}} \right]$$

- **Potential Energy of a Dipole in an External Field** Potential of a dipole in an external field can be given as,

$$U = pE(\cos\theta_1 - \cos\theta_2)$$

Here, θ_1 and θ_2 are initial and final orientations of the dipole.

- **Conductors and Insulators**

  Conductors These are those materials through which electric charge can flow easily.
  The process which involves the making of a region free from electric field is known as electrostatic shielding.

  Insulators Insulators are those materials through which electric charge cannot flow.

- **Dielectric and Polarisation**
  Dielectric Constant It is the ratio of the strength of applied electric field to the strength of reduced value of electric field on placing the dielectric between the plates of a capacitor.

  Dielectric Strength The maximum electric field that a dielectric can withstand without breakdown is called its dielectric strength.

  Polarisation The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called polarisation.

Electric Susceptibility Polarisation density of a dielectric slab is directly proportional to the reduced value of electric field. *i.e.* $P = \chi\varepsilon_0 E$; where χ is called electric susceptibility.

- **Capacitors and Capacitance** A capacitor is a system of two conductors separated by an insulating medium.

  The capacitance of the capacitor, $C = \frac{Q}{V}$

  In SI system unit of capacity is farad.

  Parallel Plate Capacitor Capacitance of a parallel plate capacitor can be given by, $C = \frac{\varepsilon_0 A}{d}$

  Effect of Dielectric on Parallel Plate Capacitor On introducing a dielectric between the parallel plates, capacitance can be given by,

$$C = \frac{\varepsilon_0 KA}{d}, \text{ where } K \text{ is the dielectric constant}$$

- **Combination of Capacitors**

  In parallel combination,

$$C_{eq} = C_1 + C_2 + \ldots\ldots + C_n$$

In series combination,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}$$

The energy U stored in a capacitor of capacitance C, with charge q and voltage V is,

$$U = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{q^2}{2C}$$

The electrostatic energy density (energy per unit volume) in a region with electric field E is

$$U = \frac{1}{2}\varepsilon_0 E^2$$

- **Common potential** It can be given as, $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

- **Loss of Energy in Sharing Charges**

  It can be given as, $\Delta U = \frac{C_1C_2}{2}\frac{(V_1 - V_2)^2}{(C_1 + C_2)}$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If 100 J of work has to be done in moving an electric charge 4 C from a place where potential is -10 V to another place where potential is V volt, find the value of V.

(a) 5 V (b) 10 V (c) 25 V (d) 15 V

2. In an electric field with $E = 0$, the potential V varies with the distance r as

(a) $V \propto \frac{1}{r}$ (b) $V \propto r$

(c) $V \propto 1/r^2$ (d) V will not depend on r

3. A car battery is charged by a 12 V supply and energy stored in it is 7.20×10^5 J. The charge passed through the battery is **CBSE 2021 (Term-I)**

(a) 6.0×10^4 C (b) 5.8×10^3 J

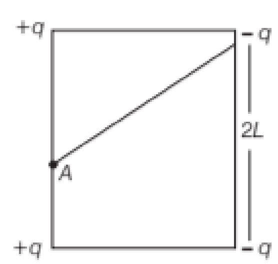
(c) 8.64×10^6 J (d) 1.6×10^5 C

4. Two charges 3×10^{-8} C and -2×10^{-8} C located 15 cm apart. At what point on the line joining the two charges is the electric potential zero?

(a) 9 cm (b) 45 cm

(c) 18 cm (d) Both (a) and (b)

5. Four charges $-q, -q, +q$ and $+q$ are placed at the corners of a square of side $2L$ is shown in figure. The electric potential at point A mid-way between the two charges $+q$ and $+q$ is **CBSE 2021 (Term-I)**



(a) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{2L}\left(1-\frac{1}{\sqrt{5}}\right)$ (d) zero

6. The electric potential V at any point (x, y, z) is given by $V = 3x^2$, where x is in metres and V in volts. The electric field at the point (1 m, 0, 2 m) is **CBSE 2021 (Term-I)**

(a) 6 V/m along $-X$-axis

(b) 6 V/m along $+X$-axis

(c) 1.5 V/m along $-X$-axis

(d) 1.5 V/m along $+X$-axis

7. Which of the following is not the property of equipotential surface? **CBSE SQP (Term-I)**

(a) They do not cross each other.

(b) The rate of change of potential with distance on them is zero.

(c) For a uniform electric field, they are concentric spheres.

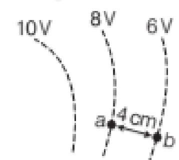
(d) They can be imaginary spheres.

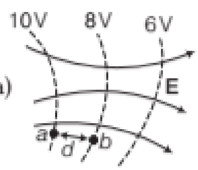
8. Equipotentials at a large distance from a collection of charges, whose total sum is not zero are **CBSE 2021 (Term-I)**

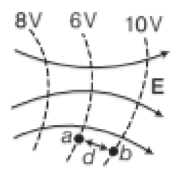
(a) spheres (b) planes

(c) ellipsoids (d) paraboloids

9. Three equipotential surfaces are shown in figure. Which of the following is correct one for the corresponding field lines?



(a)


(b)


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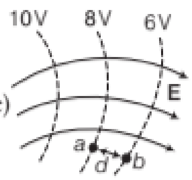


(d) None of these

10. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard

S_1 At any point inside the sphere, electric intensity is zero.

S_2 At any point inside the sphere, the electrostatic potential is 100V.

Which of the following is a correct statement? **NCERT Exemplar**

(a) S_1 is true but S_2 is false

(b) Both S_1 and S_2 are false

(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2

(d) S_1 is true, S_2 is also true but the statements are independent

11. Two charges $14\,\mu C$ and $-4\,\mu C$ are placed at (– 12 cm, 0, 0) and (12 cm, 0, 0) in an external electric field $E = \left(\frac{B}{r^2}\right)$, where $B = 1.2\times 10^6$ N/cm^2 and r is in m. The electrostatic potential energy of the configuration is **CBSE 2021 (Term-I)**

(a) 97.9 J (b) 102.1 J

(c) 2.1 J (d) – 97.9 J

12. A + 3.0 nC charge Q is initially, at a distance of $r_1 = 10$ cm from a + 5.0 nC charge q fixed at the origin. The charge Q is moved away from q to a new position at $r_2 = 15$ cm. In this process, work done by the field is **CBSE 2021 (Term-I)**

(a) 1.29×10^{-5} J (b) 3.6×10^{5} J

(c) -4.5×10^{-7} J (d) 4.5×10^{-7} J

13. On bringing an electron near to other electron, the potential energy of the system

(a) decreases (b) increases

(c) remains same (d) becomes zero

14. An electric dipole of length 1cm is placed with the axis making an angle of 30° to an electric field of strength 10^4 N/C. If it experiences a torque of $10\sqrt{2}$ Nm, the potential energy of the dipole is

(a) 0.245 J (b) 2.45 J

(c) 24.5 J (d) 245.0 J

15. What is the value of capacitance if a very thin metallic plate is introduced between two parallel plates of area A and separated at distance d ?

(a) $\varepsilon_0 A/d$ (b) $\frac{2\varepsilon_0 A}{d}$ (c) $\frac{4\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{2d}$

16. A parallel plate capacitor has a uniform electric field (Vm^{-1}) in the space between the plates. If the distance between the plates is d(m) and area of each plate is A(m^2), the energy (joule) stored in the capacitor is

(a) $\frac{1}{2}\varepsilon_0 E^2$ (b) $\varepsilon_0 EAd$

(c) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (d) $E^2 Ad/\varepsilon_0$

17. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from $2\,\mu F$ to $X\,\mu F$, the decrease in energy of the capacitor is 2×10^{-2} J. The value of X is **CBSE 2021 (Term-I)**

(a) $1\,\mu F$ (b) $2\,\mu F$

(c) $3\,\mu F$ (d) $4\,\mu F$

18. Two parallel plate capacitors X and Y, have the same area of plates and same separation between plates. X has air and Y with dielectric of constant 2, between its plates. They are connected in series to a battery of 12 V. The ratio of electrostatic energy stored in X and Y is **CBSE SQP (Term-I)**

(a) 4 : 1 (b) 1 : 4

(c) 2 : 1 (d) 1 : 2

19. If the charge on each plate of a capacitor of $60\,\mu F$ is 3×10^{-8}C. Then, energy stored in the capacitor will be

(a) 2.5×10^{-15} J (b) 1.5×10^{-14} J

(c) 3.5×10^{-13} J (d) 7.5×10^{-12} J

20. Three capacitors $2\,\mu F$, $3\,\mu F$ and $6\,\mu F$ are joined in series with each other. The equivalent capacitance is **CBSE SQP (Term-I)**

(a) $1/2\,\mu F$ (b) $1\,\mu F$

(c) $2\,\mu F$ (d) $11\,\mu F$

21. A capacitor plates are charged by a battery with V volts. After charging battery is disconnected and a dielectric slab with dielectric constant K is inserted between its plates, the potential across the plates of a capacitor will become **CBSE SQP (Term-I)**

(a) zero (b) $V/2$

(c) V/K (d) KV

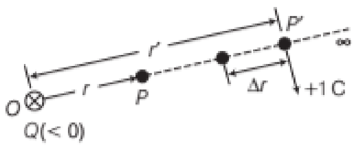
ASSERTION AND REASON

Directions (Q. Nos. 22-32) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

22. **Assertion** Work done by the electrostatic force in bringing the unit positive charge from infinity to the point P is positive.



Reason For $Q < 0$, the force on unit positive charge is attractive, so that the electrostatic force and the displacement (from infinity to P) are in the same direction.

23. **Assertion** A and B are two conducting spheres of same radius. A being solid and B hollow. Both are charged to the same potential. Then, charge on A = charge on B.

Reason Potential on both are same.

24. **Assertion** There is no potential difference between any two points on the equipotential surface.

Reason No work is required to move a test charge on the equipotential surface from one point to other.

25. **Assertion** The expression of potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$, is unaltered whatever way the charges are brought to the specified locations.

Reason Path-independence of work for electrostatic force.

26. **Assertion** An electron has a high potential energy when it is at a location associated with a more negative value of potential and a low potential energy when at a location associated with a more positive potential.

Reason Electrons move from a higher potential region to lower potential region.

CBSE SQP (Term-I)

27. **Assertion** In the absence of an external electric field, the dipole moment per unit volume of a polar dielectric is zero.

Reason The dipoles of a polar dielectric are randomly oriented.

28. **Assertion** Polar molecules have permanent dipole moment.

Reason In polar molecules, the centre of positive and negative charges coincides even when there is no external field.

29. **Assertion** Charge on all the condensers connected in series is the same.

Reason Capacitance of capacitor is directly proportional to charge on it.

30. **Assertion** An electron moves from a region of lower potential to a region of higher potential.

Reason An electron has a negative charge.

31. **Assertion** A parallel plate capacitor is connected across a battery through a key. A dielectric slab of dielectric constant K is introduced between the plates. The energy which is stored becomes K times.

Reason The surface density of charge on the plate remains constant or unchanged.

32. **Assertion** If three capacitors of capacitances $C_1 < C_2 < C_3$ are connected in parallel, and in series then their equivalent capacitances.

$C_p > C_s$

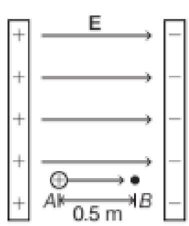
Reason $\frac{1}{C_p} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

CASE BASED QUESTIONS

Directions (Q.Nos. 33-34) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

33. **Proton in an Electric Field**

Potential difference (ΔV) between two points A and B separated by a distance x, in a uniform electric field **E** is given by $\Delta V = -Ex$, where x is measured parallel to the field lines. If a charge q_0 moves from A to B, the change in potential energy (ΔU) is given as $\Delta U = q_0\Delta V$. A proton is released from rest in uniform electric field of magnitude 8.0×10^4 Vm^{-1} directed along the positive X-axis. The proton undergoes a displacement of 0.50 m in the direction of E.



Mass of a proton $= 1.66 \times 10^{-27}$ kg

and charge on a proton $= 1.6 \times 10^{-19}$ C.

With the help of the passage given above, choose the most appropriate alternative for each of the following questions.

(i) As the proton moves from A to B, then
(a) the potential energy of proton decreases
(b) the potential energy of proton increases
(c) the proton loses kinetic energy
(d) total energy of the proton increases

(ii) The change in electric potential of the proton between the points A and B is
(a) 4.0×10^{4} V (b) -4.0×10^{4} V
(c) 6.4×10^{-19} V (d) -6.4×10^{-19} V

(iii) The change in electric potential energy of the proton for displacement from A to B is
(a) -6.4×10^{-19} J (b) 6.4×10^{-19} J
(c) -6.4×10^{-15} J (d) 6.4×10^{-15} J

(iv) The velocity (v_B) of the proton after it has moved 0.50 m starting from rest is
(a) 1.6×10^{8} ms^{-1} (b) 2.77×10^{6} ms^{-1}
(c) 2.77×10^{4} ms^{-1} (d) 1.6×10^{6} ms^{-1}

(v) If in place of charged plates, two similar point charges of 1 μC are kept in air at 1m distance from each other. Then, potential energy is
(a) 1 J (b) 1eV
(c) 9×10^{-3} J (d) zero

34. Electrostatic Potential Energy

Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.

By definition, work done in carrying charge from ∞ to any point is

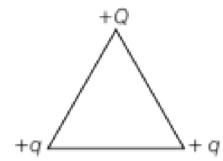
$$W = \text{Potential} \times \text{Charge}$$

This work is stored in the system of two point charges in the form of electrostatic potential energy U of the system.

(i) Work done in moving a charge from one point to other inside a uniformly charged conducting sphere is
(a) always zero
(b) non-zero
(c) may be zero
(d) None of the above

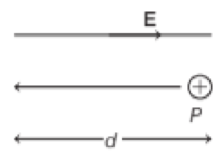
(ii) A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge
(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

(iii) Three charges are placed at the vertex of an equilateral triangle of side l as shown in figure. For what value of Q, the electrostatic potential energy of the system is zero?



(a) $-q$ (b) $q/2$
(c) $-2q$ (d) $-q/2$

(iv) In the figure, proton moves a distance d in a uniform electric field E as shown in the figure. The work done on the proton by electric field is



(a) negative (b) positive
(c) zero (d) None of these

(v) Two similar positive point charges each of 2μC have been kept in air at 3m distance from each other. What will be the potential energy?
(a) 1 J (b) 1 eV
(c) 12×10^{-3} J (d) zero

VERY SHORT ANSWER Type Questions

35. Determine the work done in moving a test charge q through the distance 1 cm along the equatorial axis of an electric dipole.

36. Why there is no work done in moving a charge from one point to another on an equipotential surface? **Foreign 2012**

37. Draw the equipotential surfaces due to an isolated point charge. **CBSE 2019**

38. Draw equipotential surface for an electric dipole. **CBSE 2019**

39. A proton released from rest in an electric field, will start moving towards a region of .... potential in the field. **CBSE 2020**

40. Depict equipotential surfaces due to an electric dipole. **CBSE 2020**

41. A charge particle $(+q)$ moves in a uniform electric field **E** in the direction opposite to **E**. What will be the effect on its electrostatic potential energy during its motion? **CBSE 2020**

42. Assume a charge starting at rest on an equipotential surface is moved off that surface and then is eventually returned to the same surface of rest after a round trip. How much work did it take to do this? Explain.

43. Do electrons tend to go to regions of high potential or low potential?

44. A proton is released at rest in a uniform electric field. Does the proton's electric potential energy increase or decrease?

    Does the proton move towards a location with a higher or lower electric potential?

45. What is the net charge on a charged capacitor?

46. Two circular metal plates, each of radius 10 cm, are parallel to each other at a distance of 1mm. What kind of capacitor do they make? Mention one application of this capacitor.

47. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?

SHORT ANSWER Type Questions

48. Draw three equipotential surfaces corresponding to a field that uniformly increase in magnitude but remains constant along x-direction.

    How are these surfaces different from that of a constant electric field along x-direction?

49. Two point charges $5\,\mu C$ and $-5\,\mu C$ are placed at points A and B, 5 cm apart.
    (i) Draw the equipotential surface of the system.
    (ii) Why do equipotential surfaces get close to each other near the point charge.

50. Deduce an expression for the potential energy of a system of two point charges q_1 and q_2 located at positions r_1 and r_2, respectively in an external field (**E**). **CBSE SQP (Term-I)**

51. The plates in a parallel plate capacitor are separated by a distance d with air as the medium between the plates. In order to increase the capacity by 66% a dielectric slab of dielectric constant 5 is introduced between the plates. What is the thickness of dielectric slab?

52. A parallel plate capacitor with air as dielectric is charged by a DC source to a potential V. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant K. State with a reason, how does
    (i) electric field between the plates and
    (ii) energy stored in the capacitor change?

53. A slab of material of a dielectric constant K has the same area as that of plates of a parallel plate capacitor but has the thickness $2d/3$, where d is separation between the plates.

    Find the expression of the capacitance when the slab is inserted between the plates of the capacitor.

LONG ANSWER Type I Questions

54. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin wire. Find the new charge density on the bigger sphere.

55. (a) Draw the equipotential surfaces corresponding to a uniform electric field in the z-direction.
    (b) Derive an expression for the electric potential at any point along the axial line of an electric dipole. **CBSE 2019**

56. (a) Draw equipotential surfaces corresponding to the electric field that uniformly increases in magnitude along with the z-direction.
    (b) Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$. What is the electrostatic potential at the points $(0, 0, \pm z)$ and $(x, y, 0)$? **CBSE 2019**

57. (a) Two point charges $+Q_1$ and $-Q_2$ are placed r distance apart. Obtain the expression for the amount of work done to place a third charge Q_3 at the mid-point of the line joining the two charges.

(b) At what distance from charge $+Q_1$ on the line joining the two charges (in terms of Q_1, Q_2 and r) will this work done be zero?

CBSE 2020

58. (a) Two point charges q_1 and q_2 are kept at a distance of r_{12} in air. Deduce the expression for the electrostatic potential energy of this system.

(b) If an external electric field (E) is applied on the system, write the expression for the total energy of this system. **CBSE 2020**

59. Define the following.

(i) Polarisation

(ii) Electric susceptibility (λ)

(iii) Electrostatic shielding

60. Choose the statement as wrong or right and justify.

(i) Inside a conductor, electric field is not zero because electrostatic potential is constant.

(ii) On insertion of dielectric, capacitance of capacitor increases.

(iii) When capacitors are connected in parallel, the amount of charge in each capacitor will be same.

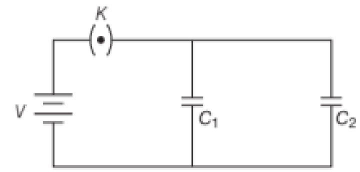
61. A parallel plate capacitor has capacitance C_0 in the absence of a dielectric. A slab of dielectric material of dielectric constant ε_r and thickness $d/3$ is inserted between the plates. What is the new capacitance when the dielectric is present?

62. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 = C_2/2$ are connected across a battery of V volts as shown in the figure. Initially, the key (K) is kept closed to fully charge the capacitors.

The key is now thrown open and a dielectric slab of dielectric K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of

(i) the net capacitance and

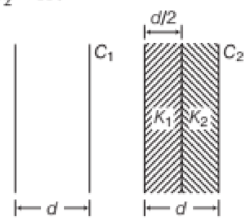
(ii) the energies stored in the combination before and after the introducing dielectric slab.



63. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in figure.

(i) Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d.

(ii) What is the value of capacitance if $K_1 = K_2 = K$?



64. Derive an expression for the energy stored in a parallel plate capacitor. On charging a parallel plate capacitor to a potential V, the spacing between the plates is halved and a dielectric medium of $\varepsilon_r = 20$ is introduced between the plates, without disconnecting the DC source. Explain, using suitable expressions, how the (i) capacitance, (ii) energy density of the capacitor changes?

LONG ANSWER Type II Questions

65. (i) Establish the relation between electric field and electric potential at a point.

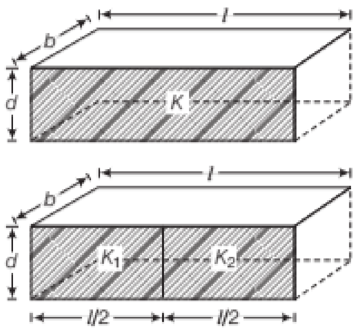
(ii) Draw the equipotential surface for an electric field pointing in $+z$-direction with its magnitude increasing at constant rate along $-z$-direction. **CBSE SQP (Term-I)**

66. (i) Depict the equipotential surfaces for a system of two identical positive point charges placed at distance d apart.

(ii) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points with positions r_1 and r_2, respectively in presence of external electric field **E**.

67. (i) Obtain the expression for the potential due to an electric dipole of dipole moment **p** at a point x on the axial line.

(ii) Two identical capacitors of plate dimensions $l \times b$ and plate separation d have dielectric slabs filled in between the space of the plates as shown in figure.



Obtain the relation between dielectric constants K, K_1 and K_2. **All India 2013**

68. (i) Describe briefly the processs of transferring the charge between the two plates of a capacitor when connected to battery. Derive an expression for the energy stored in a capacitor.

(ii) A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another unchanged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor. **CBSE 2019**

NUMERICAL PROBLEMS

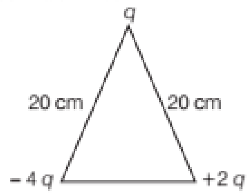
69. Calculate the potential at a point P due to a charge of 5×10^{-7} C located 11 cm away.

70. A hollow metal sphere of radius 7 cm is charged such that potential on its surface is 20 V. What is the potential at the centre of the sphere?

71. When reaching for door handle often sliding across a car seat on a dry winter day, you get a spark when your finger tip is 5 mm away from the handle. What was the potential difference between you and the door handle just before the spark? Given that dielectric strength of air $= 3 \times 10^6$ V/m.

72. Two point charges $5Q$ and Q are separated by 1m in air. At what point on the line joining the charges, is the electric field intensity zero? Also, calculate the electrostatic potential energy of the system of charges, taking the value of charge, $Q = 4 \times 10^{-7}$ C.

73. Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown in the figure.
Here, $q = 1.6 \times 10^{-10}$ C. **CBSE 2020**



74. Read the following passage and answer the question below it:

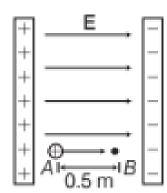
Potential difference (ΔV) between two points A and B separated by a distance x, in a uniform electric field **E** is given by $\Delta V = -Ex$, where x is measured parallel to the field lines. If a charge q_0 moves from A to B, the change in potential energy (ΔU) is given as $\Delta U = q_0 \Delta V$. A proton is released from rest in uniform electric field of magnitude 8×10^4 V/m directed along the positive X-axis. The proton undergoes a displacement of 0.50 m in the direction of E.

Mass of a proton $= 1.66 \times 10^{-27}$ kg and charge on a proton $= 1.6 \times 10^{-19}$ C.

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions.

(i) What will happen to the potential energy of proton, when it moves from A to B?

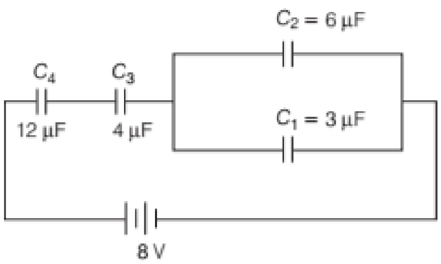
(ii) What will be the velocity (v_B) of the proton after it has moved 0.50 m starting from rest?



75. A 200 μF parallel plate capacitor having plate separation of 5 mm is charged by a 100 V DC source. It remains connected to the source. Using an insulated handle, the distance between the plates is doubled and a dielectric slab of thickness 5 mm and dielectric constant 10 is introduced between the plates. Explain with reason, how the (i) capacitance, (ii) electric field between the plates and (iii) energy density of the capacitor will change. **CBSE 2019**

76. A 100 μF parallel plate capacitor having plate separation of 4 mm is charged by 200 V DC. The source is now disconnected. When the distance between the plates is doubled and a dielectric slab of thickness 4 mm and dielectric constant 5 is introduced between the plates, how will (i) its capacitance, (ii) the electric field between the plates and (iii) energy density of the capacitor get affected? Justify your answer in each case. **CBSE 2019**

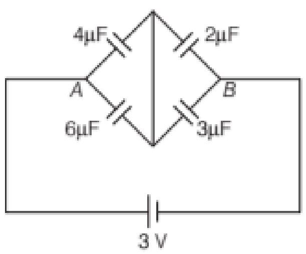
77. In a network, four capacitors C_1, C_2, C_3 and C_4 are connected as shown in the figure.



(a) Find the net capacitance of the circuit.

(b) If the charge on the capacitor C_1 is 6 μC, (i) calculate the charge on the capacitors C_3 and C_4 and (ii) net energy stored in the capacitors C_3 and C_4 connected in series. **CBSE 2019**

78. Find the total charge stored in the network of capacitors connected between A and B as shown in figure:



CBSE 2020

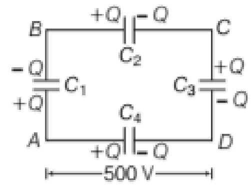
79. You are given three capacitors of 2 μF, 3 μF, 4 μF, respectively.

(a) Form a combination of all these capacitors of equivalent capacitance $\frac{13}{3}$ μF.

(b) What is the maximum and minimum value of the equivalent capacitance that can be obtained by connecting these capacitors? **CBSE 2020**

80. What is the area of the plates of a 2.5 F parallel plate capacitor, given that the separation between the plates is 0.2 cm? (You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minimum separation between the conductors). **NCERT**

81. A network of four 10μF capacitors is connected to a 500V supply as shown in the figure. Determine the equivalent capacitance of the network along AD.



82. If two parallel plate capacitors A and B are connected in series combination with the same supply voltage of V volt, the capacitor A has air in between its plates while B has dielectric of dielectric constant 4, then

(i) determine the capacitance of each capacitor, if the equivalent capacitance of the combination is 4μF.

(ii) Find the ratio of electrostatic energy stored in B to A.

ANSWERS

1. (d)	2. (d)	3. (a)	4. (d)	5. (a)
6. (a)	7. (c)	8. (a)	9. (c)	10. (c)
11. (a)	12. (d)	13. (b)	14. (c)	15. (a)
16. (c)	17. (a)	18. (c)	19. (d)	20. (b)
21. (c)	22. (a)			

23. (a) A and B are two conducting spheres of same radius. A being solid and B hollow. Both are charged to the same potential. Then, charge on A = Charge on B. Because potentials on both are same.

24. (a) The proof of this statement is simple. There is no potential difference between any two points on the surface and no work is required to move a test charge on the surface because work done

= potential difference × charge.

25. (a) The potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$ is unaltered whatever way the charges are brought to the specified locations, because of path independence of work for electrostatic force.

26. (c) Electrons move from a lower potential region to higher potential region.

27. (a) There are polar and non-polar dielectric materials. The molecules of a polar dielectric have a permanent dipole moment. However, due to random orientations net dipole moment is zero. If there is no external electric field, there is no polarisation.

28. (c) The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centre of positive and negative charges coincides. On the other hand, a polar molecule is one is which the centres of positive and negative charges are separated, even when there is no external field. Such molecules have a permanent dipole moment.

29. (c) Let two capacitors be connected in series. If $+q$ charge is installed on left plate of the first capacitor, then $-q$ charge is induced on right plate of this capacitor. This charge comes from electron drawn from the left plate of second capacitor. Thus, there will be equal charge $+q$ on the left plate of second capacitor and $-q$ charge induced on the right plate of second capacitor. Thus, each capacitor has same charge (q) when connected in series. Capacitance is quantity dependent on construction of capacitor and independent of charge.

30. (a) Electric field is set up from higher potential to lower potential. An electron is negatively charged and moves opposite to the direction of electric field, *i.e.*, from lower potential to higher potential.

31. (c) The reason is false as $\sigma' = \frac{q'}{A} = \frac{C'V'}{A} = \frac{(KC)V}{A}$

$$= \frac{Kq}{A} = K\sigma$$

(as $C' = KC$, $V' = V$ and $CV = q$)

32. (c) Assertion is true as capacitance in parallel is greater than capacitance is series. Reason is false as $C_p = C_1 + C_2 + C_3$.

33. (i) (a) Potential energy of the proton decreases as it moves in the direction of the electric field.

(ii) (b) $\Delta V = -E\Delta x = -(8.0\times 10^4 \text{ V/m})(0.50 \text{ m})$

$$= -4\times 10^4 \text{ V}$$

(iii) (c) $\Delta U = q_0\Delta V = (1.6\times 10^{-19}\text{C})(-4.0\times 10^4\text{V})$

$$= -6.4\times 10^{-15} \text{ J}$$

(iv) (b) As, $\Delta K = -\Delta U = 6.4\times 10^{-15}$ J

(from conservation of energy)

$$\Delta K = \frac{1}{2}mv_B^2$$

or $$v_B = \sqrt{\frac{2\Delta K}{m}}$$

$$= \sqrt{\frac{2(6.4\times 10^{-15}\text{J})}{(1.66\times 10^{-27}\text{ kg})}}$$

$$= 2.77\times 10^6 \text{ ms}^{-1}$$

(v) (c) Electrostatic potential energy of the system,

$$u = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{9\times 10^9\times 1\times 10^{-6}\times 1\times 10^{-6}}{1}$$

$$= 9\times 10^{-3} \text{ J}$$

34. (i) (a) Since, $E = 0$ inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface.

(ii) (c) The positively charged particle experiences electrostatic force along the direction of electric field, i.e. from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

(iii) (d) Potential energy of the system,

$$U = \frac{KQq}{l} + \frac{Kq^2}{l} + \frac{KqQ}{l} = 0$$

$$\Rightarrow \quad \frac{Kq}{l}\times[(Q + q + Q)] = 0 \Rightarrow Q = -q/2$$

(iv) (a) Since, the proton is moving against the direction of electric field, so work is done on the proton against electric field. It implies that electric field does negative work on the proton. Again, proton is moving in electric field from low potential region to high potential region hence, its potential energy increases.

(v) (c) Electric potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$

Here, $q_1 = q_2 = 1\,\mu\text{C} = 1\times 10^{-6}$ C,

$r = 1$ m and $\frac{1}{4\pi\varepsilon_0} = 9\times 10^9 \text{ N-m}^2/\text{C}^2$

$$\therefore \quad U = \frac{9\times 10^9\times 2\times 10^{-6}\times 2\times 10^{-6}}{3}$$

$$= 12\times 10^{-3} \text{ J}$$

35. Electrostatic potential at any point on the equatorial plane of dipole is zero.

$\therefore$ Work done, $\Delta W = q\Delta V = 0$

36. Change in potential is zero on an equipotential surface.

37. Refer to text and diagram on page 68 [Equipotential Surfaces in Different Cases (Case I)]

38. Refer to text and figure on page 68 [Equipotential Surfaces in Different Cases (Cave IV)]

39. decreasing

As, electric field lines starts from higher potential and ends at lower potential, so when a proton is released from rest in the field, then it moves towards the region of decreasing potential in the field.

40. Refer to text on page 68.

41. When a positive charge q moves in a direction opposite to the direction of electric field, the work done by field is negative and electrostatic potential energy increases.

42. Work done, $\Delta W = q\Delta V = q(0) = 0$

43. High potential

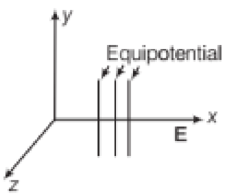
44. Proton moves from a location of higher potential to lower potential. Thus, potential energy decreases.

45. Zero

46. Parallel plate capacitor. It is used to store electrostatic energy.

47. Increases

48.

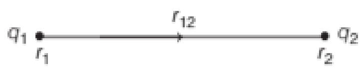


The equipotential surface are plane parallel to y-z plane. As the field is increasing in magnitude along X-axis, so the spacing between the planes decreases on moving along X-axis. But in case of constant electric field, the planes are spaced equally.

49. Refer to Q. 21 on page 75.

50. Suppose q_1 and q_2 charges are brought from infinity at locations r_1 and r_2, respectively in an external electric field.

Let $V(\mathbf{r}_1)$ and $V(\mathbf{r}_2)$ be the potentials at positions r_1 and r_2 due to external electric field $\mathbf{E}$. In this case, work is done in bringing these charges against their own electric fields and external electric field.



Work done in bringing q_1 from infinity to r_1 is,

$$W_1 = q_1 V(\mathbf{r}_1)$$

Similarly, for q_2, work done, $W_2 = q_2 V(\mathbf{r}_2)$

Work done on q_2 against the electric field due to q_1,

$$W_3 = \int_{\infty}^{r_{12}} \mathbf{F}_{12}\cdot d\mathbf{r} = \frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r_{12}} \frac{q_1 q_2}{r^2}(-d\mathbf{r})$$

$$= -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_{\infty}^{r_{12}} \frac{1}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\mathbf{r}_{12}}$$

where, $\mathbf{r}_{12} = |\mathbf{r}_2 - \mathbf{r}_1|$

$\therefore$ Potential energy of system V = Work done in assembling the configuration $= W_1 + W_2 + W_3$

$$= q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\mathbf{r}_{12}}$$

51. $\frac{\Delta C}{C} = \frac{C' - C}{C} = \frac{KC - C}{C}$ **[Ans.** $(d/2)$**]**

52. Refer to text on pages 88 and 92.

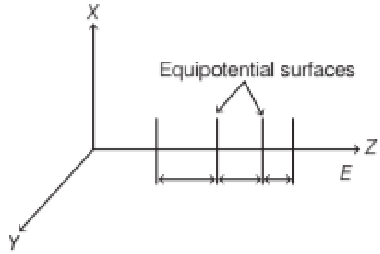
53. Refer to Q. 23 on page 95. $\left[\textbf{Ans.}\ \left(\frac{3K}{K+2}\right)\cdot\left(\frac{\varepsilon_0 A}{d}\right)\right]$

54. Refer to Q. 37 on page 77. **[Ans.** $5\sigma/6$**]**

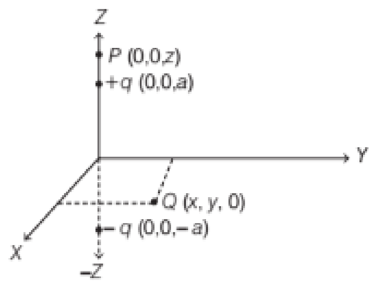
55. (a) Refer to Sol. 33 (ii) on page 80.

(b) Refer to text on page 67.

56. (a) The equipotential surface are plane parallel to X-Y plane. As the field is increasing in magnitude, the spacing between surfaces decreases.



(b) Let $P(0, 0, z)$ and $Q(x, y, 0)$ are two points on which electric potential are to be calculated.



Then, electrostatic potential at P

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-a)} - \frac{q}{(z+a)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q\times 2a}{(z^2 - a^2)}\right]$$

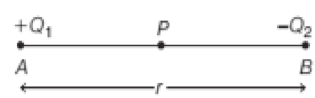
$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{(z^2 - a^2)} \qquad [\because p = q\times 2a]$$

The electrostatic potential at Q is

$$V_Q = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + y^2 - a^2}} - \frac{q}{\sqrt{x^2 + y^2 - a^2}}\right]$$

$$= 0$$

57. (a) The charges $+Q_1$ and $-Q_2$ are placed at A and B respectively, as shown



Let P be the mid-point of line joining A and B. The potential at P due to charge $+Q_1$ is,

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r/2}$$

and due to charge $-Q_2$ is

$$V_2 = \frac{-Q_2}{4\pi\varepsilon_0 r/2}$$

The resultant potential at P is

$$V = V_1 + V_2 = \frac{Q_1}{4\pi\varepsilon_0 r/2} - \frac{Q_2}{4\pi\varepsilon_0 r/2}$$

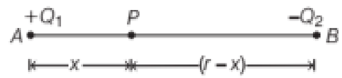
$$= \frac{2}{4\pi\varepsilon_0 r}(Q_1 - Q_2)$$

The work done to place a charge Q_3 at P is,

$$W = Q_3 V = \frac{2Q_3(Q_1 - Q_2)}{4\pi\varepsilon_0 r}$$

or $$W = \frac{Q_3(Q_1 - Q_2)}{2\pi\varepsilon_0 r}$$

(b) Let x be the distance from charge $+Q_1$ on the line joining the two charges, at which the work done will be zero as shown.



So, the potential due to charge $+Q_1$ at A is equal to potential due to charge $-Q_2$ at B at point P, i.e.

$$V_{Q_1} = V_{Q_2}$$

$$\Rightarrow \frac{Q_1}{4\pi\varepsilon_0 x} = \frac{-Q_2}{4\pi\varepsilon_0(r-x)} \Rightarrow \frac{Q_1}{x} = -\frac{Q_2}{r-x}$$

$$\Rightarrow rQ_1 - xQ_1 = -Q_2 x$$

or $$x = \frac{rQ_1}{(Q_1 - Q_2)}$$

58. (a) Refer to text on page 70.

(b) Refer to text on page 73.

59. Refer to text on pages 85 and 86.

60. (i) Refer to text on page 85.

(ii) Refer to text on page 88.

(iii) Refer to text on page 90.

61. Refer to Q. 23 on page 95.

62. Refer to Q. 29 page 96.

63. Refer to Q. 26 page 96.

64. Refer to text on page 92.

(i) and (ii) Refer to Q. 38 on page 97.

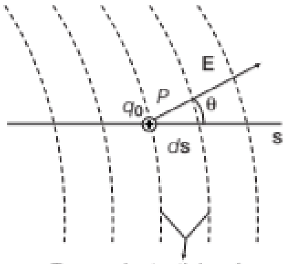
65. (i) **Relation between Electric Field and Electric Potential**

Let us consider a positive test charge (q_0) moves a distance (ds) from one equipotential surface to another.The displacement (ds) makes an angle (θ) with the direction of the electric field $(\mathbf{E})$.

Suppose a positive test charge (q_0) moves through a differential displacement ds from one equipotential surface to the adjacent surface.

We know that, the work done by the electric field on the test charge during its movement is $-q_0\, dV$. We see that, the work done by the electric field may also be written as the scalar product $(q_0 \mathbf{E} \cdot d\mathbf{S})$ or

$q_0 E \cos\theta\, ds$.



Two equipotential surfaces

(ii) Equating these two expressions for the work yields

$$-q_0 dV = q_0 E \cos\theta\, ds$$

$$\Rightarrow E\cos\theta = -\frac{dV}{ds}$$

Since, $E\cos\theta$ is the component of $\mathbf{E}$ in the direction of $d\mathbf{s}$, therefore

$$E_s = -\frac{\partial V}{\partial s}$$

$$E = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$$

where E_x, E_y and E_z are the x, y and z-components of E at any point.

$$\because \quad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

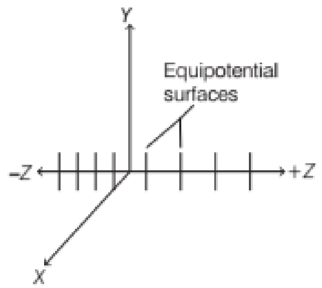
$$\therefore \quad E = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

For the simple situation in which the electric field $\mathbf{E}$ is uniform.

$$E = -\frac{\Delta V}{\Delta s}$$

Negative sign shows that the direction of electric field $\mathbf{E}$ in the direction of decreasing potential.

The equipotential surfaces for an electric field in $+z$-direction are shown below



As, the distance between the surfaces is decreasing towards $-z$-direction, this means the electric field is increasing in magnitude towards $-z$-direction at a constant rate.

66. (i) Refer to text on page 68 (Case II).

(ii) Refer to text on page 73.

67. (i) Refer to text on page 67.

(ii) Refer to Q. 24 on page 96.

68. (a) Refer to text on page 87 (Parallel Plate Capacitor) and page 92 (Energy stored in a Capacitor)
(b) Refer to Sol. 27 on page 101.

69. $V = \frac{q}{4\pi\varepsilon_0 r} = \frac{9\times10^9 \times 5\times10^{-7}}{11\times10^{-12}} = 40.9$ kV

70. Potential, $V = 20$ V

71. As, $E = \frac{dV}{dr}$

$\Rightarrow \quad 3\times10^6 = \frac{\Delta V}{5\times10^{-3}}$

$\Rightarrow \quad \Delta V = 15000\,\text{V}$

72. Refer to Example 6 on page 65 and use $U = \frac{kq_1q_2}{r}$

73. Refer to Example 15 on page 72.

74. (i) Refer to text on page 73.
(ii) Apply

$\frac{1}{2}mv^2 = eV$

$\Rightarrow \quad v = 2.77\times10^6$ m/s

75. Given, $C = 200\,\mu$F, $d = 5$ mm, $t = 5$ mm, $V = 100$ V

(i) $C = \frac{\varepsilon_0 A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0}$

$$A = \frac{200\times10^{-6}\times5\times10^{-3}}{8.85\times10^{-12}}$$
$$= 112.99\times10^3\ \text{m}^2$$

When $d' = 2d$, then $C' = \frac{\varepsilon_0 A}{2d - t + \frac{t}{K}}$

$$= \frac{8.85\times10^{-12}\times112.99\times10^3}{\left(10-5+\frac{5}{10}\right)\times10^{-3}}$$
$$= 181.8\times10^{-6} = 181.8\,\mu\text{F}$$

(ii) Charge on capacitor, $q = C_0V_0$
$$= 200\times10^{-6}\times100$$
$$= 2\times10^{-2}\ \text{C}$$
$\Rightarrow \quad C_0V_0 = C'V'$

or $\quad V' = \frac{C_0V_0}{C'} = \frac{2\times10^{-2}}{181.8\times10^{-6}} = 110$ V

$$E_0 = \frac{V_0}{d} = \frac{100}{5\times10^{-3}} = 20\times10^3\ \text{V/m}$$
$$E' = \frac{V'}{2d} = \frac{110}{10\times10^{-3}} = 11\times10^3\ \text{V/m}$$

(iii) $\overline{U} = \frac{1}{2}\varepsilon_0E_0^2 = \frac{1}{2}\times8.85\times10^{-12}\times(20\times10^3)^2$

$$= 1770\times10^{-6}\ \text{J/m}^3$$

$$(\overline{U})' = \frac{1}{2}\times\varepsilon_0(E')^2$$
$$= \frac{1}{2}\times8.85\times10^{-12}\times(11\times10^3)^2$$
$$= 535.42\times10^{-6}\ \text{J/m}^3$$

76. Given $C = 100\,\mu$F, $d = 4$ mm,
$t = 4$ mm, $V = 200$ V

(i) $d' = 2d$, $k = E_r = 5$

$$C = \frac{E_0 A}{d}$$
$$\Rightarrow 100\times10^{-6} = \frac{8.85\times10^{-12}\times A}{4\times10^{-3}}$$
$$\Rightarrow \quad A = 45.2\times10^3\ \text{m}^2$$
$$C' = \frac{E_0A}{2d - t + \frac{t}{k}}$$
$$= \frac{8.85\times10^{-12}\times45.2\times10^3}{\left(8-4+\frac{4}{5}\right)\times10^{-3}}$$
$$= 83.33\,\mu\text{F}$$

(ii) Charge on capacitor, when 200 V is applied
$$q = C_0V_0 = 100\times10^{-6}\times200 = 2\times10^{-2}\text{C}$$
Even after the battery is removed, the charge of 2×10^{-2} C on the capacitor plate remains same.

So, $C_0V_0 = C'V'$

$$\Rightarrow V' = \frac{C_0V_0}{C'} = \frac{2\times10^{-2}}{83.33\times10^{-6}} \approx 240\ \text{V}$$
$$E_0 = \frac{V_0}{d} = \frac{200}{4\times10^{-3}} = 50\times10^3\ \text{V/m}$$
$$E' = \frac{V'}{2d} = \frac{240}{2\times4\times10^{-3}}$$
$$= 30\times10^{-3}\ \text{V/m}$$

(iii) $\overline{U} = \frac{1}{2}\varepsilon_0E_0^2 = \frac{1}{2}\times8.85\times10^{-12}\times(50\times10^3)^2$
$$= 11067\times10^{-6}\ \text{J/m}^3$$
$$(\overline{U})' = \frac{1}{2}\varepsilon_0(E')^2$$
$$= \frac{1}{2}\times8.85\times10^{-12}\times(30\times10^3)^2$$
$$= 3982.5\times10^{-6}\ \text{J/m}^3$$

77. C_1 and C_2 are in parallel combination, so
$$C' = C_1 + C_2 = 3 + 6 = 9\,\mu\text{F}$$
Now, C', C_3 and C_4 are in series, so net capacitance is
$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{9} + \frac{1}{4} + \frac{1}{12}$$
$$= \frac{16}{36} \Rightarrow C = \frac{9}{4}\,\mu\text{F}$$

(b) (i) Given, $Q_1 = 6\,\mu C$

Now, potential across C_1,

$$V = \frac{Q_1}{C_1} = \frac{6}{3} = 2\,V$$

Thus charge on C_2,

$$Q_1 = C_2 V = 6 \times 2 = 12\,\mu C$$

Total charge on C_1 and C_2, $Q = 12 + 6 = 18\,\mu C$

As charge is same in series combination,

$\therefore$ Charge on C_3 and C_4 is $18\,\mu C$ each.

(ii) Total capacitance,

$$\frac{1}{C''} = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$$

$\Rightarrow \quad C'' = 3\,\mu F$

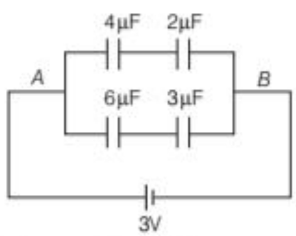
Thus, total energy stored in them is

$$U = \frac{1}{2}\frac{Q^2}{C''}$$

$$= \frac{1}{2}\frac{(18)^2}{3} \times 10^{-6}$$

$$= 54 \times 10^{-6}\,J$$

78. As the given network is like a balanced Wheatstone bridge, so no current flows through the middle wire and the network becomes as shown



So, equivalent capacitance of upper arm (series combination) is

$$C_1 = \frac{4 \times 2}{4 + 2} = \frac{8}{6} = \frac{4}{3}\mu F$$

and of lower arm is

$$C_2 = \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\mu F$$

The net capacitance of the network is

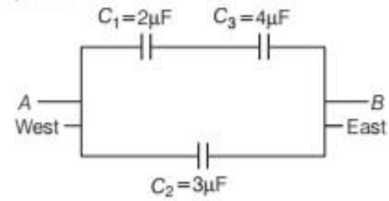
$$C_{net} = C_1 + C_2 \text{ (for parallel combination)}$$

$$= \frac{4}{3} + 2 = \frac{10}{3}\mu\,F$$

$\therefore$ Total charge, $Q = C_{net} \times V$

$$= \frac{10}{3} \times 3 = 10\mu C$$

79. (a) When C_1 and C_3 are in series and C_2 is in parallel as shown, then



$$C_{eq} = \frac{2 \times 4}{2 + 4} + 3 = \frac{4}{3} + 3 = \frac{13}{3}\mu F$$

(b) Maximum value of the equivalent capacitance is obtained when all capacitors are connected in parallel combination,

i.e. $\quad C_{eq} = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9\mu F$

or $\quad C_{eq} = C_{max} = 9\mu F$

and minimum value is obtained, when all capacitors are connected in series combination,

i.e. $$C_{eq} = \frac{C_1 \times C_2 \times C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

$$= \frac{2 \times 3 \times 4}{2 \times 3 + 3 \times 4 + 4 \times 2}$$

or $$C_{eq} = C_{min} = \frac{12}{13}\mu F$$

80. Refer to Example 2 on page 87. **[Ans.** 560 km^2]

81. $$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_{eq.} = C' + C_4 = 13.3\,\mu F$$

82. Refer to Q. 53 on page 98.

(i) $5\,\mu F$, $20\,\mu F$ (ii) $1:4$